An Advanced 3D Ray Launching Method for Wireless Propagation Prediction

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Abstract — For radio propagation prediction, recent simulations involving ray tracing offer unprecedented accuracy [1], [11], [13], [14]. These techniques surpass statistical channel models and provide a bounty of additional information including RMS delay spread, angle of arrival, and overall wideband channel impulse response. In particular, three dimensional ray tracing produces an accurate, deterministic channel model for wireless system design. This paper presents a new 3D ray tracing technique of unprecedented speed and accuracy. Simulation results are compared to microcell measurements at 1900 MHz.

I. INTRODUCTION

Three main areas of error in a ray tracing simulation are propagation modeling errors, database errors, and kinematic errors. Propagation modeling errors come from the geometrical optics models used to describe radio wave behavior. Database errors stem from the limitations of a finite, numerical description of the world. The ability of a ray tracing algorithm to find and interpret radiation paths determines the kinematic errors of a simulation. This paper focuses on a method to launch and interpret rays that model the wavefronts of radio propagation; we present a new method that eliminates many of the kinematic errors associated with a ray launching scheme.

This paper concentrates on three major ray launching concepts. It first establishes methods for geodesic ray launching, originally proposed by Seidel [13], [12]. The following section reviews the principles and inconsistencies of interpreting ray information using the reception sphere model found in the literature [11]. Finally, the paper introduces a new method for interpreting ray information using a special weighting function to construct ray-traced wavefronts. The distributed wavefront model has several important advantages over the reception sphere models used in previous literature (i.e. [13], [15]). This paper provides fundamental principles to construct an extremely accurate three dimensional ray launcher for propagation prediction, unlike any previously proposed.

II. PROPAGATION AND DATABASE MODELING

An environment database for 3D ray launching consists of flat polygons that represent the surfaces of buildings and terrain. Flat polygons are easy to describe numerically and allow simple reflection calculations. The well-defined edges also lend themselves to diffraction calculations [6]. A polygon database may be constructed easily using computer-aided design (CAD) software. Figure 1 shows an example of a ray-traced environment in AutoCAD® [5].

![Fig. 1. An example of rays traced through an urban environment (Rosslyn, VA).](image)

A key issue for implementing widespread, site-specific propagation tools is the formal specification of environment databases. These databases must contain all of the information relevant to a propagation simulation such as geometry, material properties, and surface characteristics. For this reason, current research is working to develop the Real World Database Format (RWDF), a standard method for constructing a site-specific propagation database.

III. RAY TRACING TECHNIQUES

An alternative to ray launching that avoids the geometrical complexities of a ray launching algorithm is the method of images. The method of images uses image theory to place artificial sources in the environment that model reflections from the flat planes of a database [3], [7]. Because image theory determines exact radiation paths, the method of images introduces no errors into the radiation paths it finds. However, this method only works for reflected modes of propagation, since diffraction introduces infinite degrees of freedom in the direction of a ray path.

The primary drawback to image theory algorithms is their heavy dependence on the number of elements in the environment database. For single reflection, each
additional surface in the database can double the number of images available to a receiver. Therefore, image theory should not be used to render complicated, three-dimensional cityscapes, which require multiple reflections from hundreds of surfaces. Image theory is best suited for simplified, two-dimensional city blocks.

A versatile ray tracing technique, and the one explored in the following sections, launches rays from a transmitter and reflects them through the environment. For this method, the launched rays that pass arbitrarily close to a receiver establish the actual radiation paths. Although this paper concentrates on reflection, ray launching techniques are extendible to diffraction. The principle advantage of this method is its ability to quickly render a complicated 3D scene since the algorithm has only a linear dependence on the number of database elements. The overall processing time can be reduced further by incorporating spatial discrimination, such as a bounding volume hierarchy [2].

IV. GEODESIC RAY LAUNCHING

In a ray launching scheme, rays emanate from a unit sphere centered on the transmitter location. Launch points around this sphere follow a regular, computer-generated geometry. There are two desirable characteristics of a ray launch geometry:

- **Large-Scale Uniformity** - The launch points must distribute evenly around the sphere so that all regions of space are illuminated equally by rays. Large-scale uniformity delivers unbiased ray coverage in three dimensions.
- **Small-Scale Uniformity** - The local pattern of rays impinging on a wavefront should be a predictable, uniform pattern. This corresponds to equal angles between a ray and its neighbors, which assists the interpretation of wavefront information.

Ray shooting from the vertices of regular polyhedrons is the only way to exactly satisfy the two uniformity criteria [15]. Since no regular polyhedron has more than twenty vertices, high precision ray tracing must use other geometries [12].

A. Geodesic Geometry

The geodesic sphere arises by tessellating the faces of a regular polyhedron and extrapolating the intersection points to the surface of a sphere [4]. Figure 2 depicts the geodesic facets and vertices that result when the sides of an icosahedron are subdivided into smaller equilateral triangles. The geodesic vertices provide ray launch points with equivalent angular separation around the entire sphere [13]. Moreover, each ray will have exactly six neighbor rays that surround the original in a predictable hexagonal pattern. The receiver schemes presented in this paper exploit this regularity when calculating power levels.

B. Geodesic Aberrations

Uniformity allows for simple interpretation of a ray launch. The reception sphere model takes advantage of the uniformity of a geodesic launcher to collect rays using a simple circular area. Because the geodesic sphere approximates uniformity, there are some discrepancies in angular separation among the launched rays. For instance, each ray emanating from one of the twelve vertices of the icosahedron has only five neighboring rays rather than the usual six (see Figure 3). This type of aberration becomes insignificant for heavily tessellated spheres [12].

Another aberration occurs in the angular separation between rays. There is an average discrepancy of twenty percent between the smallest and largest angular separations on a sphere [15]. Angular separation plays an important part in a ray tracing calculation. It determines the distance to the closest ray on the same wavefront - vital information for interpreting simulation results. Regardless of the tessellation frequency, there will be a twenty percent variation between the minimum and maximum separation on a geodesic sphere. High tessellation frequency, however, does mitigate the variations in localized angular separation [12]. Characterizing the angular separation for each ray should then use the maximum separation angle with respect to its six neighbors, rather than a single separation value for all geodesic rays.

Despite aberrations, an average value for angular separation is a useful parameter in a simulation. The angular resolution of a geodesic sphere is a function of the tessellation frequency $N$, the number of triangles on the edge of an icosahedron face. Assuming that the $20N^2$ total geodesic facets are congruent equilateral triangles, Equation (1) finds the average radial separation by equating...
their total area with the $4\pi$ steradians of a unit sphere.

$$\alpha = \frac{1}{N} \sqrt{\frac{4\pi}{5\sqrt{3}}} = \frac{1.205}{N} \text{ radians} = \frac{69.0^\circ}{N}$$

Equation (1) is an excellent approximation to the average angular spreading of rays. It gives a measure of angular resolution for a three dimensional ray tracing simulation.

V. THE RECEPTION SPHERE

Once rays are traced through a scene, a program must interpret the results by measuring voltage or power levels at arbitrary points in space. A common method to interpret the traced ray information is the reception sphere model. This method assumes uniform ray launching from a geodesic sphere in an environment composed of flat surfaces, such as buildings or planar segments of terrain. The simple assumptions and easy implementation of the reception sphere model make it a useful ray tracing algorithm.

A. Reception Sphere Mechanics

The reception sphere model surrounds a receiver point with a sphere of varying size. Rays that intersect this sphere contribute to the total received power. One of the traits of the reception sphere model is its simple implementation since the sphere-ray intersection tests are easy to describe mathematically.

The size of the reception sphere depends on the characteristics of the incoming ray. Because rays spread out as they leave the source, the reception sphere must increase size accordingly. Figure 4 shows a two dimensional example of fitting reception sphere sizes with incoming rays to guarantee the collection of exactly one ray from each wavefront. If the sphere is too large, more than one ray from the same wavefront adds to the total power. An undersized sphere can miss a wavefront altogether.

B. Ray Double Counting

In two dimensions, reception spheres work perfectly; a third dimension adds complexity and ambiguity. Figure 5 shows the ideal impingement of geodesic-launched rays onto a spherical wavefront. The minimum radius for a reception sphere to guarantee the collection of at least one ray from a wavefront is $\frac{1}{\sqrt{3}}$, the distance between rays [11], [13]. This radius sweeps out a circular area across the wavefront where sometimes two rays fall within the sphere, registering additional voltage and power.

Fig. 5. Reception sphere double counting on a uniform wavefront. The pattern represents the ideal impingement of 3D rays launched from a geodesic sphere.

Fig. 6. Double count errors (white regions) along an ideal wavefront.

The double counting errors occur frequently across a measured wavefront. Figure 6 highlights the double counting on a map of an ideal wavefront. The probability of a randomly placed receiver experiencing a double count comes from the geometry of Figure 6. For the ideal case, double countings occur with a probability of $\frac{2\pi}{3\sqrt{3}} - 1$ or 20.9% of the time. This probability is independent of tessellation frequency and worsens with increased geodesic and reflecting surface aberrations.

Incoherent accumulation of this additional power will result in a +3 dB error. However, the arrival angles and the path lengths of two rays from the same wavefront are identical. Since their amplitudes and phases are equal, a coherent addition will result in twice as much predicted voltage and, subsequently, a +6 dB power error. Assuming random location of a receiver along a geodesic wavefront, a coherent voltage measurement with reception spheres will increase the mean power by at least 1.25 dB and introduce an additional 2.4 dB of standard deviation error. Simulations that do not use localized angular separation to calculate reception sphere sizes exhibit even worse statistics [16].

VI. DISTRIBUTED WAVEFRONTS

The method of distributed wavefronts remedies the problems inherent with the reception sphere model while maintaining the speed and simplicity of ray launching. It re-
moves the double count errors that the previous model introduced. Instead of counting hit-or-miss rays, the weighting of a nearby ray is a function of proximity to the receiver. Thus, the total received power comes from a three dimensional weighting of the nearby rays. The problem of field strength prediction reduces to matching a weighting function to the specific launch geometry.

A. Weighting Functions

The localized uniformity of a geodesic launch make it an excellent candidate for the use of weighting functions. The uniform pattern of impinging rays (see Figure 5) fits nicely into a weighting scheme. A radially symmetric weighting function must be found that, when placed on all of the nearby geodesic ray points of Figure 5 and summed, equals a constant for any position across the wavefront.

We used a Monte Carlo approach to find valid weighting functions. This method uses a tabulated weighting function for points in the Figure 5 pattern to calculate how the weighted sums vary across the potential wavefront. Brute force iterations increase or decrease the tabulated function values until the curve converges to a useful weighting function, as in Figure 7. Table I is an example of a smooth function that will weight the surrounding rays so that the total is nearly invariant of the position across the wavefront.

![Distributed Wavefront Weighting Function](image)

**Fig. 7.** A plot of the tabulated radial weighting function.

### Table I

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B. Distributed Wavefront Geometry

The curve in Figure 7 weights power or field contributions as a function of arc length from receiver to ray, shown as $d$ in Figure 8. Equations for total distance traveled by the ray, $R$, and the normalized distance along a wavefront, $x$, are given by (2) and (3).

$$R = \sqrt{L^2 + t^2 + 2Lt\cos\theta}$$  \hspace{1cm} (2)

$$x = \frac{d}{\alpha R} = \frac{1}{\alpha} \tan^{-1} \left[ \frac{t \sin \theta}{L + t \cos \theta} \right]$$  \hspace{1cm} (3)

$L$ - ray length prior to intersection
$t$ - length from receiver to intersection
$\theta$ - receiver's specular deviation angle from ray
$d$ - arc length from receiver to ray
$\alpha R$ - arc length from ray to nearest neighboring ray

The normalized distance $x$, a ratio of arc lengths, allows the distributed wavefront model to use the same weighting function given by Table I in the range $0 \leq x \leq 1$.

![Fig. 8. Geometry for a distributed wavefront (2D projection).](image)

C. Intersection Formulation

The distributed wavefront method accumulates field strength from rays if there is an unobstructed path between the receiver location and the ray’s source (either the transmitter or a surface intersection). Equation (4) shows the total electric field phasor calculated from the source points of traced rays.

$$\vec{E}_T = \sum_{i=1}^{N} f(x_i) \ V(\vec{x}_R, \vec{x}_i) \ e^{j\phi_i} \ E_i$$  \hspace{1cm} (4)

$\vec{x}_R$ - receiver location
$\vec{x}_i$ - location of $i$th ray source
$E_i$ - electric field vector associated with
$\phi_i$ - phase of the electric field
$N$ - total ray sources in environment
$f(x_i)$ - distributed wavefront weighting
$V(\vec{x}, \vec{y})$ - visibility function [value 1 if LOS, 0 if not]

For this coherent summation, $E_i$ and $\phi_i$ are functions of the total distance traveled, $R$, and the attenuations and phase shifts from surface reflections. Similar terms for diffraction and scattering can also be introduced into (4).

The environment database determines the visibility function, $V(\vec{x}, \vec{y})$, which has a value of 1 if a clear path exists.
between two vectors and 0 if the path is obstructed. A key characteristic of Equation (4) is that the source points, $x_i$, do not depend on receiver location. For this reason, a computer only needs to trace a scene once for a fixed transmitter. Characteristics for subsequent receivers only require recalculation of the visibility functions and field values for the new locations. Dividing ray tracing into separate tracing and linking stages saves calculation time for repeated simulation of a fixed transmitter.

VII. Results and Measurements

The method of distributed wavefronts was tested on a region of the Virginia Tech campus shown in Figure 9. Extensive wideband measurements were performed at this site by Saldanha in [10]. These measurements were made at 1900 MHz, using a spread spectrum channel sounder with 10ns resolution [8]. The transmitter and receiver were at heights above ground of 3 and 1.5 meters, respectively.

Figure 10 shows ray traced results using the distributed wavefront model compared to the measurements for an arbitrary receiver location. The ray tracing predicts the position and amplitudes of multipath components very well, with slight perturbations due to the accuracy of the computer database. As expected, a few multipath components were omitted in the prediction since only reflected modes of propagation were simulated in a simplified environment. Distributed wavefront ray launching proves to be an accurate model for finding a channel impulse response.

VIII. Conclusions

Geodesic spheres and distributed wavefront methods increase the accuracy of 3D ray tracing for propagation prediction without sacrificing simplicity and speed. The distributed wavefront method corrects ray double counts in the reception sphere model. The method also lends itself to spatial interpolation of discrete 3D antenna patterns. A link budget for the intersection points also reveals an important principle in ray tracing: subsequent simulations in an environment with a fixed transmitter do not require additional ray tracing. Future work will incorporate diffraction, material properties, and polarization effects.

![Image](61x363 to 262x500)

Fig. 9. Rays traced for a Virginia Tech campus database at receiver location 14.

![Graph](41x260)

Fig. 10. Predicted and measured power delay profiles (0 dBm transmit power at 1900 MHz). Total received power is -55.7 dBm measured, compared to -56.8 dBm predicted.

REFERENCES