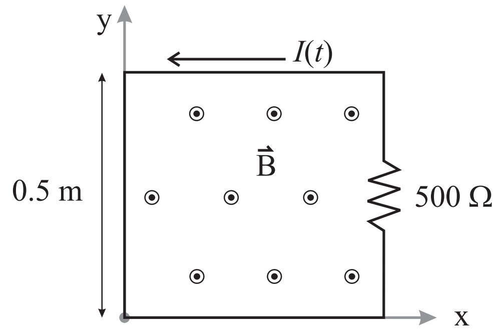
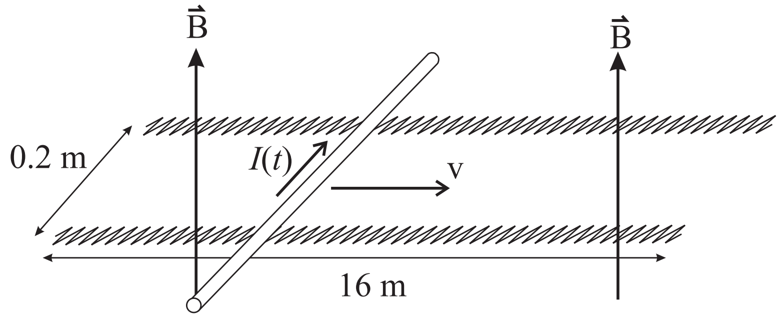
ECE 3025 Homework 11: Inductance & Faraday's Law

1. A perfectly conducting filament containing a small 500Ω resistor is formed into a square, as illustrated below. Find $I(t)$ if $\vec{B} =$ (a) $0.3\cos(120\pi t - 30°)\hat{z}$ Teslas; (b) $0.4\cos[\pi(ct - y)]\hat{z}$ μT, where $c = 3 \times 10^8$ m/s.



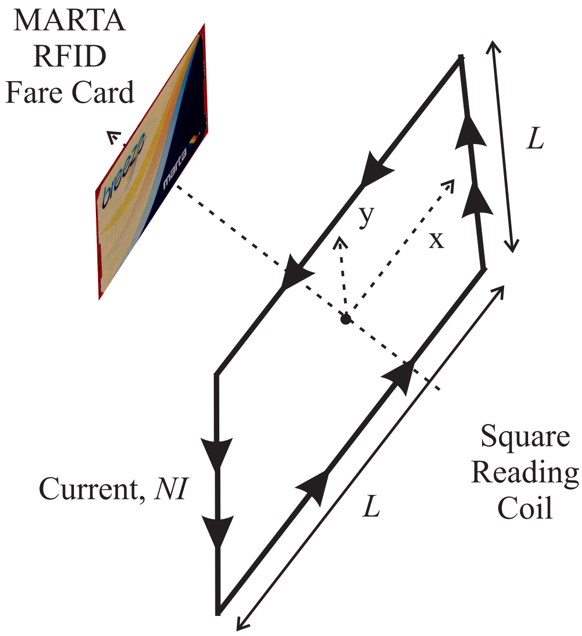
2. The rails in the figure below each have a resistance of 2.2 Ω/m. The bar moves to the right at a constant speed of 9 m/s in a uniform magnetic field of 0.8 T. Find $I(t)$, $0 < t < 1$s, if the bar is at $x = 2$m at $t = 0$ and: (a) a 0.3Ω resistor is present across the left end with the right end open-circuited; (b) a 0.3Ω resistor is present across each end.



3. Using first-principles in field theory, calculate the per-unit-length inductance and capacitance of a transmission line consisting of two parallel wires whose axes are separated by b units of length and have a radius of a units of length. Assume the current is carried on the surface of the conductive wires. Also calculate the intrinsic impedance and velocity of propagation.

4. **Inductive RFID:** Inductive *radio frequency identification* (RFID) is finding more and more applications in anti-shoplifting devices, keyless entry systems, prescription drug authentication,

and fare card tracking – to name just a few. One such application of RFID that is near-and-dear to many Georgia Tech students is the new MARTA BreezeWay system. The system replaces the old token system with an RFID fare card that keeps track of rides on the MARTA with a small computer chip that contains an *electronically erasable and programable read-only memory* (EEPROM). When a rider approaches the new BreezeWay turnstyle, the card is brought in proximity to a side panel containing a loop of wires. This scenario is illustrated below:



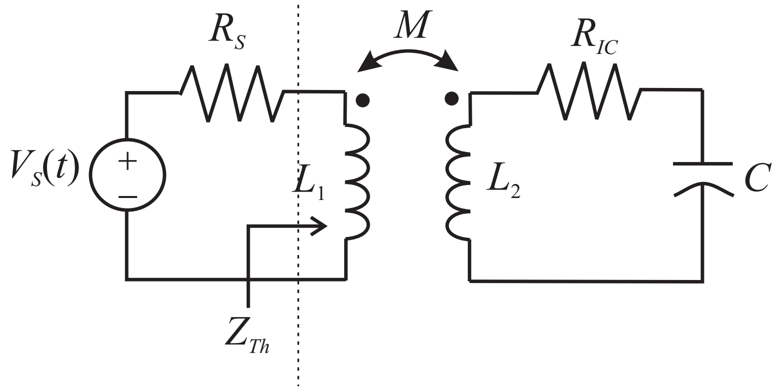
The current in the reader coil oscillates at 13.56 MHz and, following Faraday's Law, excites a voltage around the coil in the fare card. This voltage is then rectified by the chip to provide power to the memory and communication circuitry. Based on this scenario and the following specifications, answer the following questions.

Reader Coil Self-Inductance	L_1	40.0	μH
RFID Card Self-Inductance	L_2	2.8	μH
RF Integrated Circuit Resistance	R_{IC}	25	Ω
Matching Capacitor Value	C	50	pF
Source Resistance	R_S	75	Ω
Number of Card Coil Turns	N_c	6	
Number of Reader Coil Turns	N_r	20	
Square Coil Dimension	L	10	cm

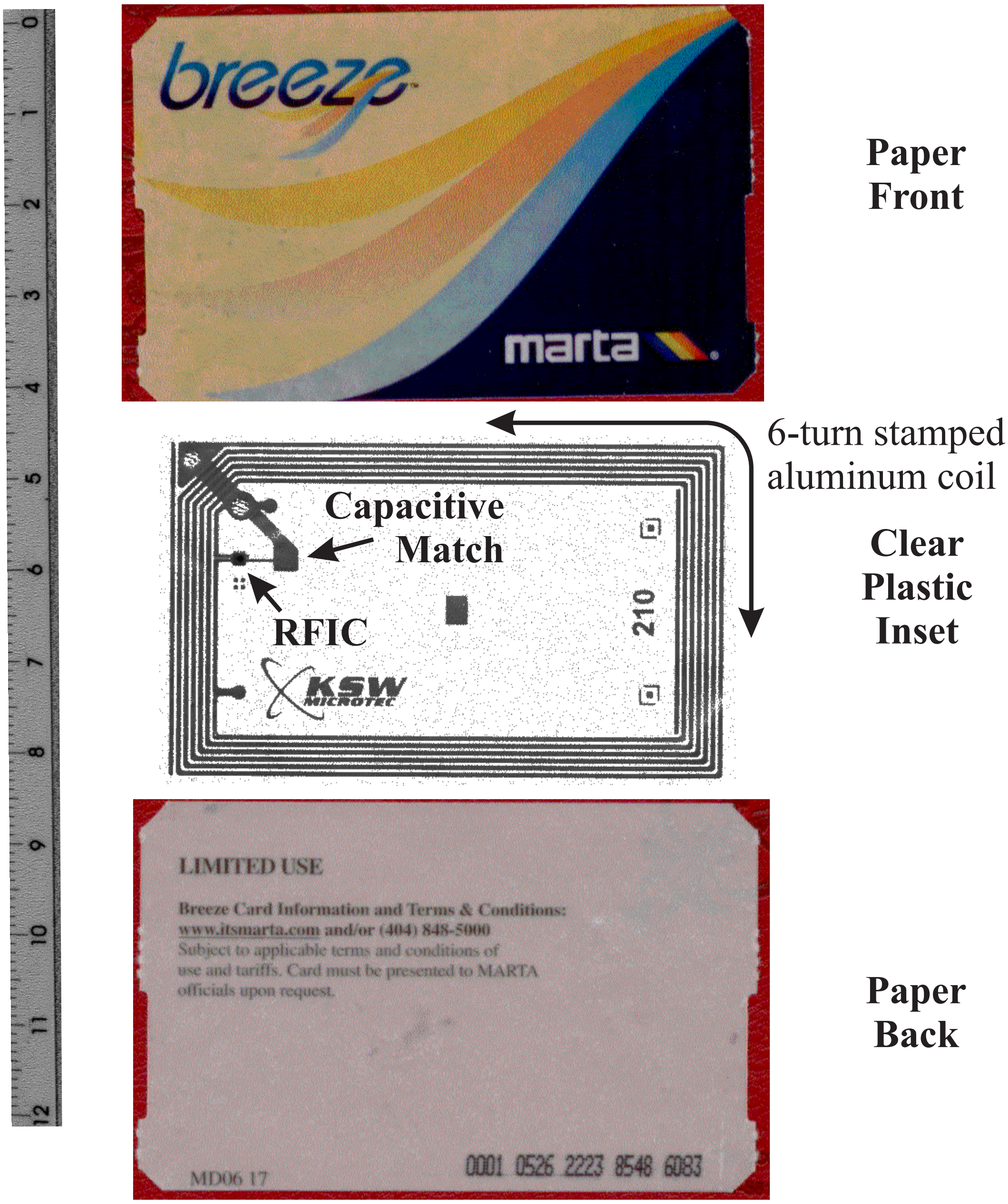
(a) Power available for coupling into an inductive RFID will be proportional to the magnitude-*squared* of the magnetic field, $\vec{H}$. Make a plot of $\|\vec{H}_z(0,0,z)\|^2$ for the square-loop reader coil, normalized against its maximum value, $\|\vec{H}_z(0,0,0)\|^2$. What does your graph tell you about the reader-tag separation range of this particular system? Note: You may use your calculation from the previous homework to complete this problem.

(b) Estimate the mutual inductance between a MARTA card that is centered on the z-axis, parallel to the reader coil, and z units away from the plane of the coil. You may approximate the magnetic fields as constant across the area of the RFID tag. Recall that mutual inductance is defined as the ratio of total flux through the coils and the current through the coils at the reader, $M = \Psi_{21}/I$.

(c) What is the Thevenin equivalent impedance of the reader circuit in the absence of a card? What is the Thevenin equivalent impedance of the reader circuit that is 5 cm away from the reader?



See following pages for additional specifications for this system.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