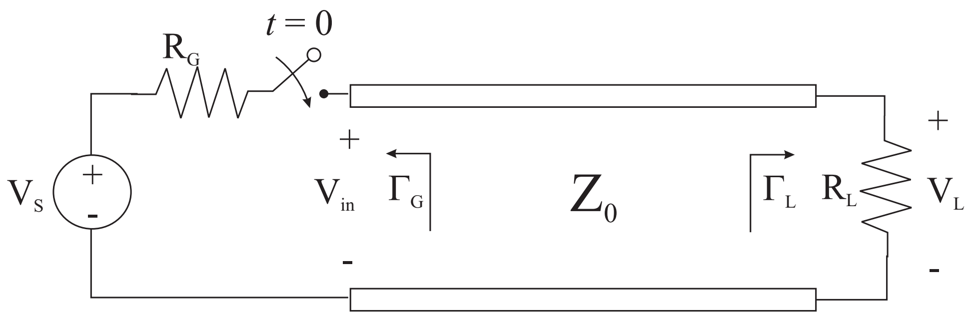
ECE 3025 Homework 2: Reflections on Transmission Lines

1. Three transmission lines are cascaded together, the first one with impedance Z_0, the second with impedance Z_1, and the third with impedance Z_2. What is the impedance of the middle section Z_1 in terms of the other two impedances that maximizes the amplitude of the initial voltage of a DC pulse traveling on the last transmission line segment? (5 points)

2. Two mismatched transmission lines are cascaded together with a switched DC-source and resistive load that are also mismatched. If the transit time of the first line is 1.2 ns and the transit time of the second line is .5 ns, write down the first 12 times that you will see voltage changes at the output if the DC switch is turned on at t=0. (5 points)

3. Use the diagram below to answer the following questions. (15 points)



   (a) If the transmission line is completely discharged before the switch is thrown, write the equation for V_{in} shortly after the switch turns on and the circuit is excited by the DC source, V_S.

   (b) It is possible to organize the reflections on a transmission line as a geometrical series. For example, the stead-state load voltage, V_L, may be written as the following series:

$$V_L = \underbrace{(1+\Gamma_L)}_{\text{load transmission}} V_{in} + \underbrace{\Gamma_L\Gamma_G(1+\Gamma_L)}_{\substack{\text{load reflection}\\ \text{source reflection}\\ \text{load transmission}}} V_{in} + \underbrace{\Gamma_L\Gamma_G\Gamma_L\Gamma_G(1+\Gamma_L)}_{\substack{\text{load reflection}\\ \text{source reflection}\\ \text{load reflection}\\ \text{source reflection}\\ \text{load transmission}}} V_{in} + \cdots$$

   Using a basic geometric series relationship ($\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ for $|r| < 1$) we concluded that the steady state transmission to the load is given by γ_L:

$$\gamma_L = \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_G}$$

What is γ_L in terms of Z_0, R_L, and R_G? Use your result in (a) to demonstrate that, in the steady state, the transmission line behaves like a pair of short circuits.

(c) Prove that the steady-state input voltage to the transmission line, V_{in}^{ss}, is related to the initial V_{in} with the exact same transmission coefficient. In other words, prove that $\gamma_G = \gamma_L$ and that the steady-state DC voltages are identical on either side of the line. Hint: start with a series similar the above expression, taking into account reflections and transmissions on the *source* side. This should have the following form:

$$V_{in}^{ss} = V_{in} + \underbrace{\qquad\qquad}_{\substack{\text{load reflection} \\ \text{source transmission}}} V_{in} + \underbrace{\qquad\qquad}_{\substack{\text{load reflection} \\ \text{source reflection} \\ \text{load reflection} \\ \text{source transmission}}} V_{in} + \cdots$$

(d) Substitute your result for (a) into your result for (c) to calculate the steady-state DC voltage on the transmission line. What equivalent circuit does this behavior represent?

(e) Explain in words what is happening to the transmission line between the initial excitation in (a) and the steady-state operation in (d).