ECE 3025 Homework 7: Fields and Charges

1. **Integral Set-up:** As you've probably noticed, most of the integrals used to compute E-field from charge distributions are fairly complicated. Any realistic problem must be evaluated by computer. However, no computer can set-up the integration for you, so it's important that you be able to do this. For the cases in this problem, set-up and simplify (but do not evaluate) the corresponding voltage or E-field integrals. (15 points)

   As an example, if there is a uniform line of charge with density ρ_L from (0,0,0) to (0,0,D), then the E-field at an arbitrary point in space is given by:

$$\begin{aligned}
\vec{E}(\vec{r}) &= \int_0^D \frac{\overbrace{\rho_L(\vec{r}')}^{\rho_L}(\vec{r}-\vec{r}')\overbrace{dL}^{dz'}}{4\pi\epsilon|\vec{r}-\vec{r}'|^3} \\
\vec{E}(x\hat{x}+y\hat{y}+z\hat{z}) &= \int_0^D \frac{\rho_L(\overbrace{x\hat{x}+y\hat{y}+z\hat{z}}^{\vec{r}}\overbrace{-0\hat{x}-0\hat{y}-z'\hat{z}}^{-\vec{r}'})dz'}{4\pi\epsilon|\underbrace{x\hat{x}+y\hat{y}+z\hat{z}}_{\vec{r}}\underbrace{-0\hat{x}-0\hat{y}-z'\hat{z}}_{-\vec{r}'}|^3} \\
\vec{E}(x,y,z) &= \int_0^D \frac{\rho_L(x\hat{x}+y\hat{y}+[z-z']\hat{z})dz'}{4\pi\epsilon(x^2+y^2+[z-z']^2)^{\frac{3}{2}}} \\
&= (x\hat{x}+y\hat{y}+z\hat{z})\frac{\rho_L}{4\pi\epsilon}\int_0^D \frac{dz'}{(x^2+y^2+[z-z']^2)^{\frac{3}{2}}} - \hat{z}\frac{\rho_L}{4\pi\epsilon}\int_0^D \frac{z'dz'}{(x^2+y^2+[z-z']^2)^{\frac{3}{2}}}
\end{aligned}$$

   This integral is ready to be fed into the computer and evaluated.

   (a) A circular disk of charge rests on the xy-plane, centered at the origin with radius R and uniform charge density ρ_S (in C/m^2). What is the E-field at the point (x, y, z)?

   (b) A spherical shell of charge has radius 1 and surface charge density of $\rho_S(x, y, z) = \rho_0\sqrt{1-z^2}$. What is the E-field at the point (x, y, z)?

   (c) Charge is placed on an infinitely long helical wire that follows the parametric equation: $z = t$, $\phi = t$, $r = R$, where R is a constant. The charge is uniformly distributed along the wire with charge density ρ_L. What is the E-field at the point (x, y, z)? Hint: you may integrate along the wire with respect to the parametric value t – but you will have to pick the proper dL; you cannot simply equate $dL = dt$ since t is not a natural unit of length. You may need to dust off your old calculus textbook for this one.

2. A 2-μC point charge is located at $A(4, 3, 5)$ in free space. Find E_ρ, E_ϕ, and E_z at $P(8, 12, 2)$. (5 points)

3. The volume charge density $\rho_v = \rho_0 \exp(-|x| - |y| - |z|)$ exists over all of free space. Calculate the total charge present. (5 points)

4. Uniform line charges of 0.4 μC/m and -0.4 μC/m are located in the $x = 0$ plane at $y = -0.6$ and $y = 0.6$m, respectively. Let $\epsilon = \epsilon_0$. Find $\vec{E}$ at: (a) $P(x, 0, z)$; (b) $Q(2, 3, 4)$. (5 points)

5. Two identical uniform line charges, with $\rho_L = 27$nC/m, are located in free space at $x = 0$, $y = \pm 0.4$m. What force per unit length does each line charge exert on the other? (5 points)