ECE 3025 Homework 9: Laplace's Equation

1. **Numerical Computation of Voltage:** To answer this question, you must download the Matlab package for Homework 8. This package contains the code for the `PlotVoltageXY` function as well as a file with many ready-made boundary condition matrices. For each case in this problem, load the boundary conditions and run `PlotVoltageXY` to generate the equipotential contour plots. Label the lines with the proper voltage, then draw streamlines with arrows to denote the direction of the E-fields. (10 points)

   Example: Load the boundary conditions for a box that is 20 units $\times$ 20 units with 0 volts around all the edges except the bottom; the bottom side of the box has a voltage of 100V. These boundary conditions are stored in `ExampleBC`. Calculate, plot, and label the equipotential lines for V = 0, 1, 2, 5, 10, 20, 30, 50, and 100V.

   What You Enter: First, load the boundary condition file that contains all of the boundary condition matrices. This is accomplished by entering

   ```
   load BoundaryConditions.mat
   ```

   This file already has the `ExampleBC` matrix. If you would like to look at a plot of these boundary conditions, enter the following code:

   ```
   imagesc(ExampleBC); axis equal; axis xy; colorbar;
   ```

   You should see the 100V boundary condition on the bottom of a square that 0 everywhere else. Now it's time to perform the voltage calculation. The syntax for `PlotVoltageXY` is fairly straightforward. The first argument is the boundary condition matrix. The second argument is an array of all the equipotential lines that you would like to plot. For this example, the following code will produce the voltage plots we want:

   ```
   V = PlotVoltageXY(ExampleBC,[100 50 30 20 10 5 2 1 0]);
   ```

   This function, if executed properly, will calculate the voltages according to Laplace's equation and will produce two plots as output. The first plot is a colorful representation of the voltages as a function of space. The second plot is a simple line diagram of the equipotential contour lines. Examples of these two plots are shown below:
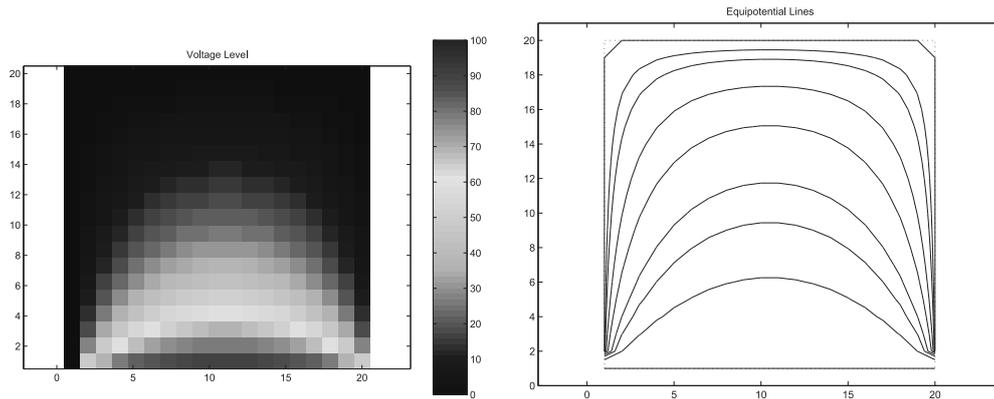
Print out the plot on the right and label the voltages on the equipotential lines. Draw at least a half-dozen streamlines (complete with directional arrows) to denote the E-fields.

Notes:

- If you have a fast computer and want to see the relaxation algorithm at work, set `FAST=false;` at the top of the `PlotVoltageXY` code. This will show an animation of the calculations being performed.

- This code is 2D and thereby assumes that all fields are independent of z. This is useful for calculating the fields due to charges on a transmission line, since we can use the xy plane to represent the cross section of E-fields around the line. All of the examples that you will work in this problem involve cross-sections of transmission lines.

- If you are interested in trying some of your own custom boundary conditions, the boundary condition matrix can be any dimension $N \times M$. If a location in the matrix represents a boundary condition, simply set that point equal to the boundary voltage. If voltage at a location is to be calculated, set that equal to the `nan` value (not-a-number) in Matlab.

- For this type of relaxation algorithm, the boundary conditions around the edge of the matrix *must* be specified. Other points in the boundary condition matrix are optional.

Complete the following:

(a) Graph equipotential contours and streamlines for a parallel-plate transmission line with the bottom plate at a voltage of -100V and the top plate at a voltage of +100V. The boundary conditions for this case are in the matrix `ParallelPlateBC`. Plot and label the equipotential lines between -100V and 100V in increments of 20V. Draw streamlines to denote E-fields.

(b) Graph the case of a two-wire transmission line with the bottom line at a voltage of -100V and the top line at a voltage of +100V. The boundary conditions for this case are in the matrix `TwoLineBC`. Plot and label the equipotential lines for 0, ±5, ±10, ±20, ±50, and ±100V. Draw streamlines to denote E-fields.

(c) Graph the case of a coaxial transmission line with the outer conductor at 0V and the inner conductor at 100V. The boundary conditions for this case are in the matrix `CoaxialBC`. Plot and label the equipotential lines between 0V and 100V in increments of 10V. Draw streamlines to denote E-fields.

(d) Graph the case of a microstrip transmission line with the ground plane at 0V and the upper strip line at 100V. The boundary conditions for this case are in the matrix `MicrostripBC`. Plot and label the equipotential lines between 0V and 100V in increments of 10V. Draw streamlines to denote E-fields.