Curriculum Topic :
Electrostatic Fields

ESF1 : Coulomb's Law in Cartesian Coordinates

Module Outline:	
Prerequisite Skills	Competencies
Supplemental Reading and Resources	Assessments
Power Point Slides and Notes	

Prerequisite Skills

Prerequisites / Requirements:
MATH 4 semesters of calculus
PHYS 2 semesters of undergraduate physics

Competencies

Competency ESF.1: **Compute electric field from a description of charge in space using the Cartesian coordinate system.**

Competency Builders:
ESF.1.1 Describe four types of charge (point charge, line, surface, and volume charge densities)
ESF.1.2 Compute total charge given an arbitrary charge distribution function
ESF.1.3 Define the electric field vector
EST.1.4 Compute electric field from an arbitrary charge distribution function in Cartesian space

Supplemental Reading and Resources

Supplemental Reading Materials:

Prof. Andrew Peterson's Lecture Notes (Fields and Waves Lectures 1 and 4)

Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

Question: ESF.1.1 *Competency:* ESF.1.1

What are the four ways to describe charge distributions in space as well as their corresponding units?

Answer:

point charge (C), line charge density (C/m), surface charge density (C/m^2), and volume charge density (C/m^3)

Question: ESF.1.2 *Competency:* ESF.1.3

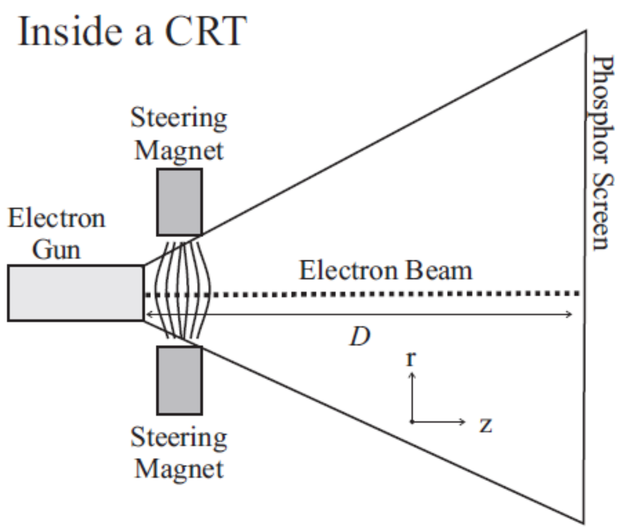
Which of the following units is incorrect for the electric field? a) N/C b) V/m c) J/C/m d) F/A

Answer:

- d -

Question: ESF.1.3 *Competency:* ESF.1.3,4

Electron Gun in a TV: (26 points) Below is a schematic of a cathode ray tube (CRT). To create images, an electron beam is swept across a phosphor screen using electromagnets. This electron beam can be approximated to be a thin, uniform line of free space charge with densisty, ρ_L. Answer the questions below based on this model.



(a) If the beam starts at the cylindrical coordinate point (0,0,0) and ends at the point $(0,0,D)$, what is the electric field as a function of space, $\vec{r}$, inside the CRT? (Set up the integral and simplify, but you do not have to evaluate the expression.) (10 points)

(b) Since the charges in the electron beam are not confined, there is a tendency of the beam to widen as it travels towards the phosphor screen. If a single electron (-1.60×10^{-19} C) is brought right very close to this beam (a small distance ρ), how much force is exerted upon it and in which direction? For close-in fields, you can approximate the line charge as infinite. (10 points)

(c) The electron gun takes static electrons and fires them in the $\hat{z}$ direction using strong electric fields. On the box in the diagram labeled "Electron Gun", place an arrow in the direction of the electric fields inside the box. (6 points)

Answer:

(a) Below is the full integral setup for the finite line of charge.

$$\vec{E}(\vec{r}) = \int_0^D \frac{\overbrace{\rho_L(\vec{r}')}^{\rho_L}(\vec{r}-\vec{r}')\overbrace{dL}^{dz'}}{4\pi\epsilon|\vec{r}-\vec{r}'|^3}$$

$$\vec{E}(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z) = \int_0^D \frac{\rho_L(\overbrace{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}^{\vec{r}}\overbrace{-0\hat{a}_x - 0\hat{a}_y - z'\hat{a}_z}^{-\vec{r}'})dz'}{4\pi\epsilon|\underbrace{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}_{\vec{r}}\underbrace{-0\hat{a}_x - 0\hat{a}_y - z'\hat{a}_z}_{-\vec{r}'}|^3}$$

$$\vec{E}(x,y,z) = \int_0^D \frac{\rho_L(x\hat{a}_x + y\hat{a}_y + [z-z']\hat{a}_z)dz'}{4\pi\epsilon(x^2+y^2+[z-z']^2)^{\frac{3}{2}}}$$

$$= (x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)\frac{\rho_L}{4\pi\epsilon}\int_0^D \frac{dz'}{(x^2+y^2+[z-z']^2)^{\frac{3}{2}}} - \hat{a}_z\frac{\rho_L}{4\pi\epsilon}\int_0^D \frac{z'dz'}{(x^2+y^2+[z-z']^2)^{\frac{3}{2}}}$$

Nearly full credit was given to students who made it to line 1. Complete credit was given for anyone who showed the explicit dependence of $\vec{r}'$ and the variable of integration z, which occurs in line 2.

(b) This follows directly from the formula on the back sheet for (approximately) infinite line charge.

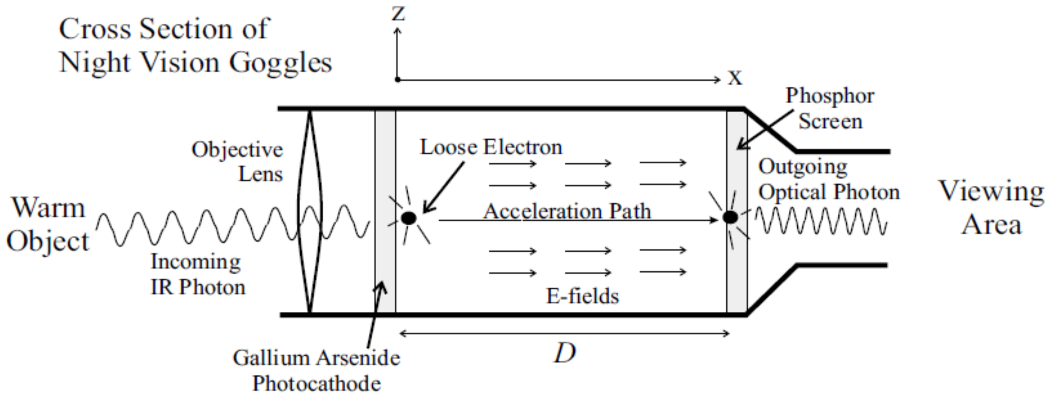
$$\vec{F}(\rho) = q\vec{E} = \frac{q\rho_L}{2\pi\rho\epsilon}\hat{a}_\rho$$

where $q = -1.60 \times 10^{-19}$. Since both q and ρ_L are negative, the force will be away from the line of charge.

(c) The arrow should point in the -z direction (left). If you reversed the arrow (sign error) you received -4. If your arrow was pointing in any other direction or was not placed in the box labeled electron gun, you received 0.

Question: ESF.1.4 *Competency:* ESF.1.2,4

(4) **Night Vision Goggles:** Commercial night vision goggles operate by receiving invisible infrared photons which, upon striking a thin Gallium-Arsenide *photocathode*, release a single electron into an acceleration chamber. In this chamber, strong electric fields accelerate the electron and smash it into a phosphor screen, where optical photons are released for the viewing person to see.



The E-field of the acceleration chamber is given by the following equation:

$$\vec{E}(\vec{r}) = 10,000 \sin\left(\pi \frac{x}{D}\right) \hat{a}_x$$

How much work is done on the electron (-1.60×10^{-19} C) before it is smashed onto the phosphor screen?

Answer:

Night Vision Goggles: The total work performed on the electron is

$$\text{Work} = -q \int_0^D \vec{E}(\vec{r}) \cdot \underbrace{d\vec{L}}_{dx\,\vec{a}_x} = -10,000q \int_0^D \sin\left(\pi \frac{x}{D}\right) dx = -\frac{20,000Dq}{\pi}$$

where $q = -1.6 \times 10^{-19}$ Coulombs.