<u>Curriculum Topic</u> :
Electrostatic Fields

<u>ESF3</u> : Gauss's Law in Integral Form

Module Outline:	
Prerequisite Skills	Competencies
Supplemental Reading and Resources	Assessments
Power Point Slides and Notes	

Prerequisite Skills

Prerequisites / Requirements:
ESF2 Coulomb's Law in Advanced Coordinate Systems

Competencies

Competency ESF.3: **Apply Gauss's Law in integral form to electrostatic problems.**

Competency Builders:
ESF.3.1 Define an electric flux density vector field
ESF.3.2 Construct a flux integral for electric field surrounding a charge distribution

Supplemental Reading and Resources

Supplemental Reading Materials:

Prof. Andrew Peterson's Lecture Notes (Fields and Waves Lectures 5 and 6)

Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

Question: ESF.3.1 *Competency:* ESF.3.1

What are the units of electric flux density?

Answer:

C/m^2

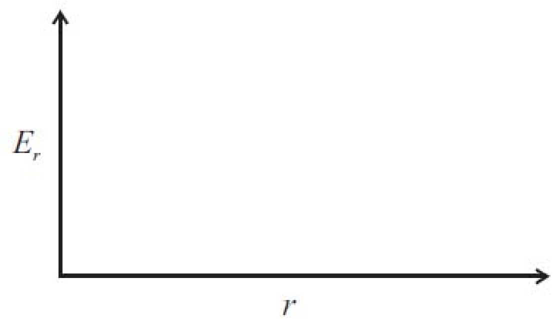
Question: ESF.3.2 *Competency:* ESF.3.2

Electrostatic Charge Distributions: All of the field distributions in this problem are free-space and may be written in the following form:

$$\vec{E}(r,\phi,\theta) = E_r(r)\hat{r}$$

Make a rough sketch in the graph provided of $E_r(r)$ for the following charge distributions.

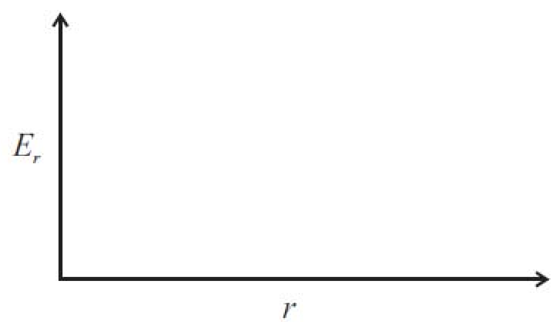
(a) At the origin is a +1 C point charge. Surrounding this charge is a spherical shell of uniform charge density centered at the origin at a radius of R and a total charge of -2 C. (**8 points**)



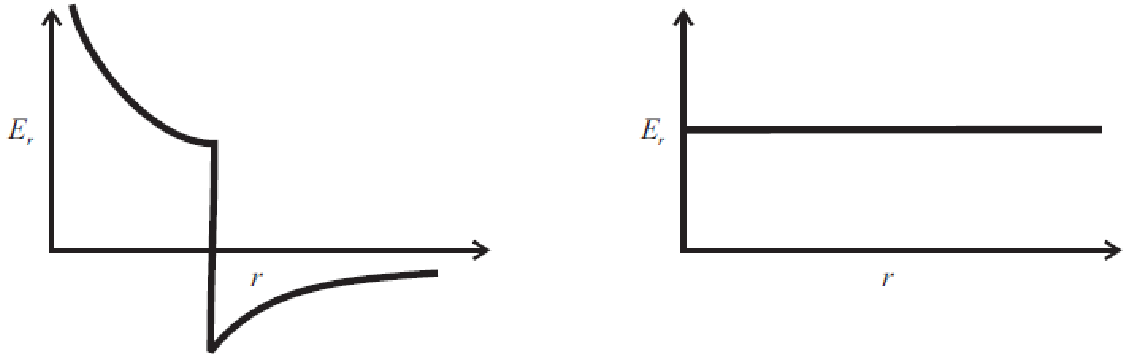
(b) There is a charge distribution in space of the form:

$$\rho_v(r,\phi,\theta) = \frac{\rho_o}{r}$$

(**8 points**)



Answer:



Question: ESF.3.3 *Competency:* ESF.3.2

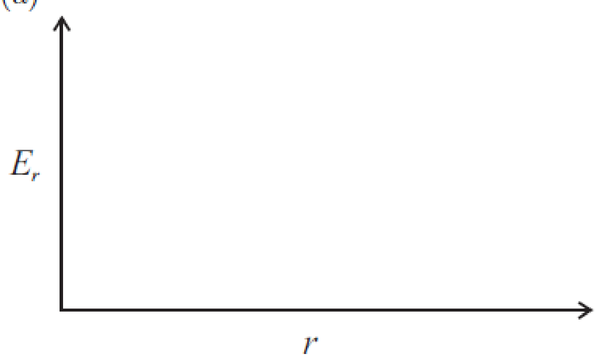
Electrostatic Charge Distributions: All of the field distributions in this problem are free-space and may be written in the following form:

$$\vec{E}(r,\phi,\theta) = E_r(r)\hat{r}$$

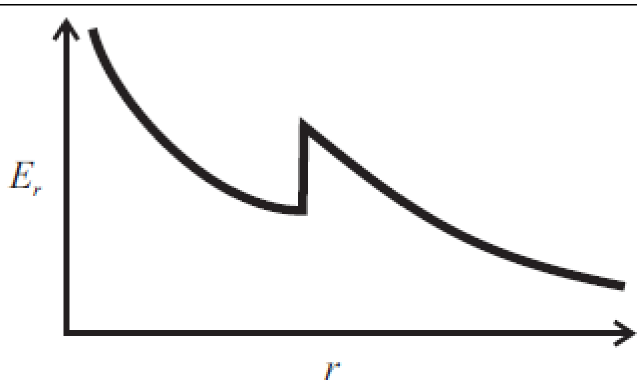
Make a rough sketch in the graph provided of $E_r(r)$ for the following charge distributions.

(a)

At the origin is a +1 C point charge. Surrounding this charge is a spherical shell of uniform charge density centered at the origin at a radius of R and a total charge of +1 C.

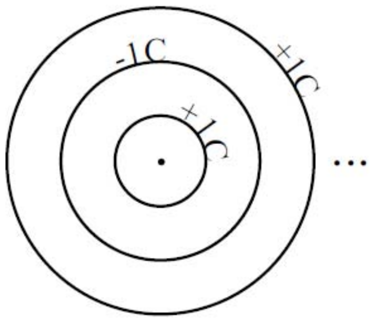


Answer:

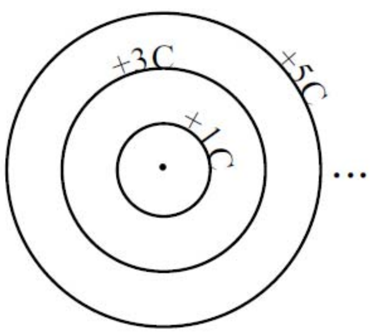


Question: ESF.3.4 *Competency:* ESF.3.2

(a) **E-Fields of Charge Shells:** Imagine a scenario of successive spherical shells of uniform charge density centered at the origin, increasing in size, and alternating +1 C and -1 C in total charge. Thus, the first shell has radius R and contains a total charge of +1 C spread out uniformly on its surface. The second shell has radius $2R$ and contains a total charge of -1 C. The third shell has radius $3R$ and total charge +1 C. This pattern continues to infinity. Sketch the magnitude of the E-field as a function of distance from the origin, r. You do not have to show exact amplitudes – only the basic behavior with respect to r.



(b) **Gauss's Law:** Imagine a scenario of successive spherical shells of uniform charge density centered at the origin, increasing in size by 1m and total charge by +2 C on every shell. Thus, the first shell has radius 1m and contains a total charge of +1 C spread out uniformly on its surface. The second shell has radius 2m and contains a total charge of +3 C. The third shell has radius 3m and total charge +5 C. This pattern continues to infinity. What is the value of the electric field very far from the origin (large r).

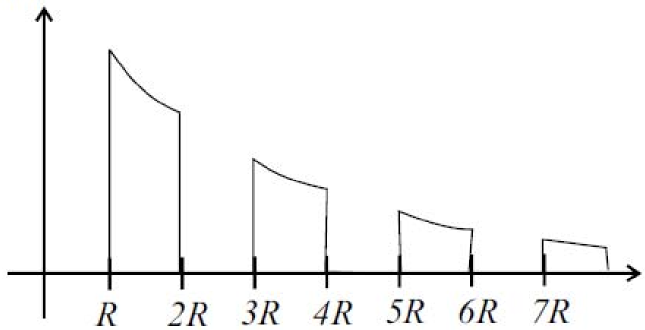


Answer:

(a) **E-Fields of Charge Shells:** Remember Gauss's law: the total charge inside determines the electric flux density coming out of an enclosed surface. In this problem there is also spherical symmetry with respect to electric flux density and, hence, the electric field. For every $nR < r < (n+1)R$ with odd n, the net total charge within a sphere of radius r is +1 C. Thus, the field will be identical to that of a +1 C point charge centered at the origin:

$$\vec{E} = \frac{+1\text{ C}}{4\pi\epsilon_0 R^2}\hat{a}_r$$

For every $nR < r < (n+1)R$ with even n, the net total charge within a sphere of radius r is 0 C; there are no fields within this region. Thus, a rough sketch of field magnitude would resemble:



(b) **Gauss's Law:** If we draw a closed spherical surface around the origin of distance $R + \delta r$ (where $\delta r <$ 1m), we can see that we will enclose a total of $+R^2$ charge. Thus, our electric field is

$$\vec{E} = \frac{R^2}{4\pi\epsilon_0 (R+\delta r)^2}\hat{a}_r$$

which becomes a constant

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\hat{a}_r$$

in the limit of large R.