# ESF3: Gauss's Law in Integral Form

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## Definition of Electric Flux Density Vector

Coulomb's Law: If we know charge, what is the field?
Gauss's Law: If we know field, what is the charge?

Useful Quantity: Electric Flux Density

$\vec{D} = \epsilon \vec{E}$ units of $C/m^2$

↑ permittivity (constant of proportionality in a simple medium)

Represents the flux of charge, independent of medium



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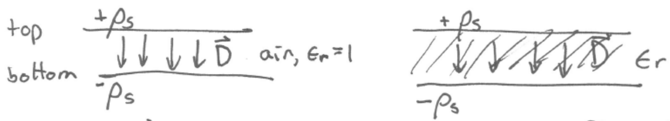
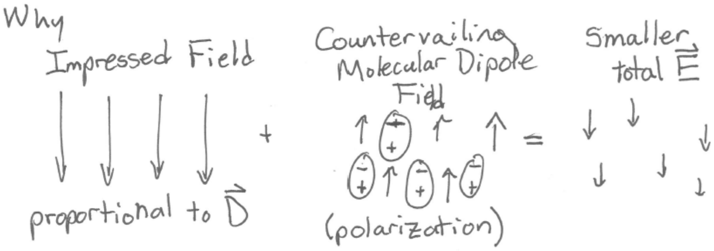
## Electric Flux Between Parallel Plates

For example, a parallel-plate capacitor with constant charge on the plates:




Same $\vec{D}$, regardless of $\epsilon_r$, but different $\vec{E}$.

Why



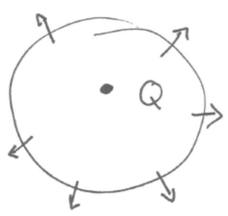



## Flux Density Around a Point Charge

Note, from Coulomb's Law for Point Charges:

$$\vec{D} = \epsilon_o \vec{E} = \frac{Q}{4\pi R^2}\hat{r}$$

Flux about a sphere




Surface Area × Uniform Flux Density = total enclosed charge

$$4\pi R^2 \times \|\vec{D}\| = Q$$

## Closed Surface Flux Integral

More Elegantly

$$\oint_S \vec{D}\cdot d\hat{n} = Q$$

$Q$ ← enclosed charge (C)

$d\hat{n}$ ← surface normal

$S$ ← enclosing surface

About a Sphere

$$\int_0^{2\pi}\int_0^{\pi} \underbrace{\left[\frac{Q\,\hat{r}}{4\pi R^2}\right]}_{\vec{D}} \cdot \underbrace{\left[R\,d\theta \cdot R\sin\theta\, d\phi\ \hat{r}\right]}_{d\hat{n}}$$



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## Gauss's Law in Integral Form

Thus

$$= \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^{\pi} \sin\theta\, d\theta}_{2} \frac{Q}{4\pi} \underbrace{(\hat{r}\cdot\hat{r})}_{=1}$$

$= Q$ total charge (note how difficult this would have been in Cartesian coordinates)

By superposition,

$$\oint_S \vec{D}\cdot d\hat{n} = \sum_{n=1}^{N} Q_n = \int_V \rho_v(\vec{r}\,')\, dv'$$

$S$ is the volume-enclosing surface

$\vec{r}\,' = x'\hat{x} + y'\hat{y} + z'\hat{z}$ $\quad dv' = dx'dy'dz'$

in Cartesian Coordinates



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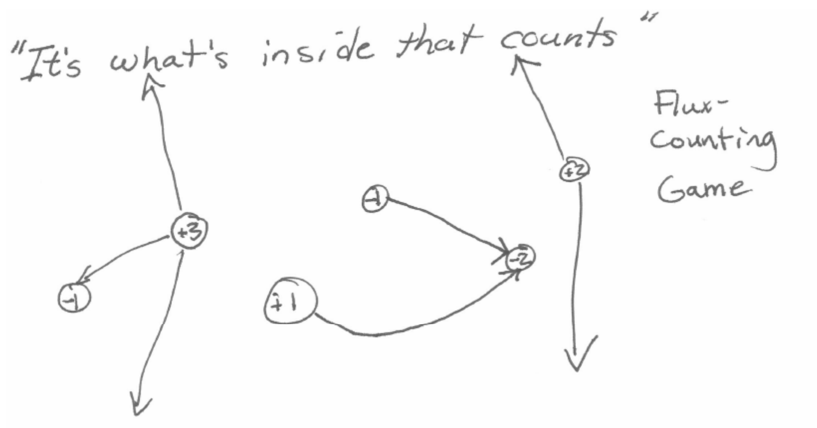
## The Flux Counting Game



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## Example: Calculating Fields Using Gauss's Law

**Charge Distributions:** All of the field distributions in this problem are free-space and may be written in the following form:

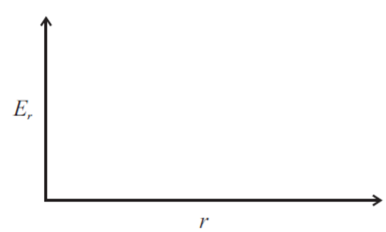
$$\vec{E}(r,\phi,\theta) = E_r(r)\hat{r}$$

Make a rough sketch in the graph provided of $E_r(r)$ for the following charge distributions.

There is a charge distribution in space of the form:

$$\rho_v(r,\phi,\theta) = \frac{\rho_0}{r^2}$$

(**8 points**)



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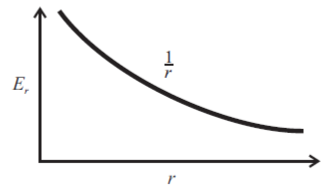
## *Solution*




Volume of enclosed charge accumulates *r*^3 volume, net *r* charge when the taper is considered

Surface area of sphere with radius *r* is proportional to *r*^2

Net drop of 1/r for E-field



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