ESF5: Scalar Electric Potential

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Scalar Electric Potential Definition

Recall

$$W = -\int_A^B \vec{F}\cdot d\vec{L} = -Q\underbrace{\int_A^B \vec{E}\cdot d\vec{L}}_{\substack{+\Delta V_{ab}\\ -\Delta V_{ba}}}$$

Zero Reference at Infinity:

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot d\vec{L}$$

path doesn't matter

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Voltage About a Point Charge

Example 1: Voltage due a point charge

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E} \cdot d\vec{L}$$

Q r

$$\vec{E}(r) = \frac{Q\,\hat{a}_r}{4\pi\epsilon_0 r^2} \qquad d\vec{L} = dr\,\hat{a}_r$$

$$V(r) = -\int_{\infty}^{r} \frac{Q\,\hat{a}_r \cdot (dr'\,\hat{a}_r)}{4\pi\epsilon_0 r'^2} = -\int_{\infty}^{r} \frac{Q\,dr'}{4\pi\epsilon_0 r'^2}$$

$$= \left.\frac{Q}{4\pi\epsilon_0 r'}\right|_{\infty}^{r} = \frac{Q}{4\pi\epsilon_0 r}$$

note $\frac{1}{r}$ fall off

Attributes About Voltage Function

– Voltage is conservative (for static electric fields)

$$\oint V(\vec{r})\,dL = 0$$

– Voltage is linear

$$V(\vec{r}) = \sum_{n=1}^{N} \frac{Q_n}{4\pi\epsilon_0 |\vec{r} - \vec{r}_n|}$$

or

$$V(\vec{r}) = \iiint_{vol} \frac{\rho_v(\vec{r}')\,dV}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$$

for space charge

Gradient Goes from Voltage to Field

– Calculating fields from Voltage

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}(\vec{r}') \cdot d\vec{L} \qquad \vec{E}(\vec{r}) = -\nabla V(\vec{r})$$

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

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Gradient of Point Charge Voltage

Let's try this on the point charge formula:

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r}$$

But this is in spherical coordinates, where the gradient operation is different:

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Recovered Field Expression

Spherical Coords:

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi$$

$$\vec{E} = -\nabla V(r) = -\left[-\frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r + 0\,\hat{a}_\theta + 0\,\hat{a}_\phi\right]$$

$$= \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r$$

Other Notes

- To find the work used to bring a charge from $\vec{A}$ to $\vec{B}$:

$$W = Q\left[V(\vec{B}) - V(\vec{A})\right]$$

We can also replace Q with current

$$\underbrace{\cancel{W}P}_{J/s} = \underbrace{I}_{C/s}\left[V(\vec{B}) - V(\vec{A})\right]$$

- In Cylindrical Coordinates

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z$$

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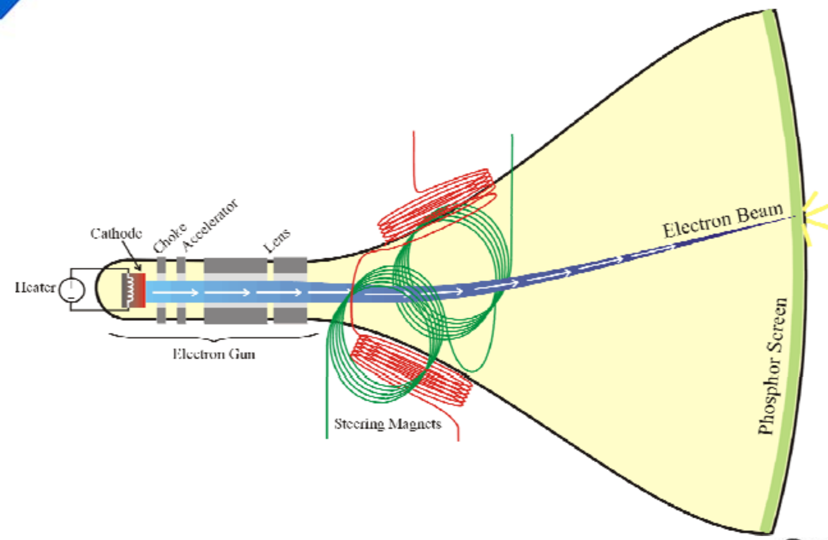

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