MSF1: Currents and Metals

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Representing Change in Charge

Current and Current Density

$I = \frac{dQ}{dt}$ C/s

1D current flow

For 3D,

$\vec{J}$ = current density $C/m^2/s$

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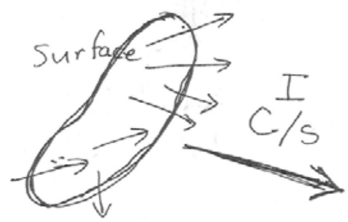
Currents in 3D

For 3D,

$\vec{J}$ = current density $C/m^2/s$

$$I = \int_S \vec{J} \cdot d' \; \hat{n}$$

S not closed



South bound I-75/85

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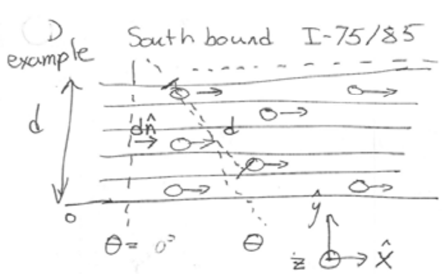
Analogy to Understand Flux

example



$\vec{J} = J_o \hat{x}$ for $x, y \leq$,

$\vec{J} \cdot d\hat{n} = J_o \cos\theta$

$I = J_o \cos\theta \; d$

$\vec{J}$

$\hat{n}$

$\vec{J} \cdot \hat{n} = \|\vec{J}\| \cos\theta$

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Continuity of Charge in Magnetostatics

$$\oint_S \vec{J}\cdot d\hat{n} = \begin{cases} 0 & \text{no charge storage} \\ \dfrac{dQ}{dt} & \text{change in charge} \end{cases}$$

$$Q = \int_{vol} \rho_v(\vec{r})\, dv \qquad \frac{dQ}{dt} = \int_{vol} \frac{\partial \rho_v(\vec{r},t)}{\partial t}\, dv$$

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Continuity Equation

Integral Form

$$\oint_S \vec{J}\cdot d\hat{n} = \int_{vol} \frac{\partial \rho_v(\vec{r},t)}{\partial t}\, dv$$

Differential form:

$$\nabla\cdot\vec{J} = \frac{\partial \rho_v(\vec{r},t)}{\partial t}$$

"Steady State DC"

$$\nabla\cdot\vec{J} = 0$$

Word Form:

Charge cannot be created, destroyed, or teleported.

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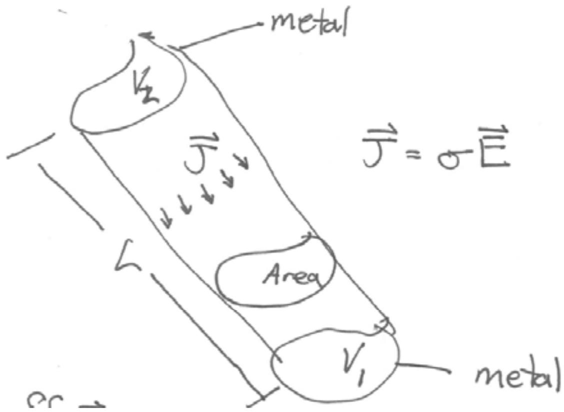
Example: Derivation of Ohm's Law



$$\vec{J} = \sigma \vec{E}$$

$$\vec{E} = -E_0 \hat{z}$$

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Current in Resistive Prism

$$\iint_A \vec{J} \cdot d\hat{n} = \iint_A \vec{E}\,\sigma \cdot d\hat{n}$$

$$I = \sigma A E_0$$

now: $$\int_0^L I\,dz = -\int_0^L \sigma A E_0\,dz$$

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Final Result is Ohm's Law

$$L I = \sigma A \underbrace{-\int_0^L E_o \, dz}_{V_2 - V_1}$$

$$V = I \underbrace{\frac{L}{\sigma A}}_{R}$$

Ohm's Law

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