MSF2: Biot-Savart Law

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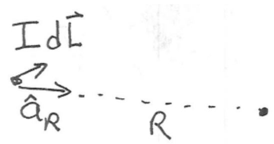
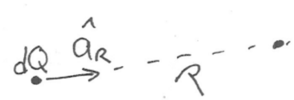
Biot-Savart vs. Coulomb's Law





$$d\vec{H} = \frac{I d\vec{L} \times \hat{a}_R}{4\pi R^2}$$

Magnetic Field

$$d\vec{E} = \frac{dQ\, \hat{a}_R}{4\pi \epsilon_o R^2}$$

Electric Field

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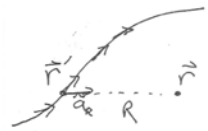
Integral Form of Biot-Savart Law

Integral Forms

$$\vec{H} = \int \frac{I\,d\vec{L} \times \hat{a}_R}{4\pi R^2}$$

Line Current



$R = \|\vec{r} - \vec{r}'\|$ (point of observation: $\vec{r}$; variable of integration: $\vec{r}'$)

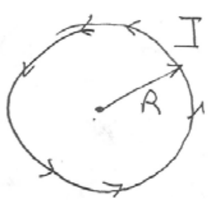
$$\hat{a}_R = \frac{\vec{r} - \vec{r}'}{\|\vec{r} - \vec{r}'\|}$$

$$\vec{H} = \int \frac{I\,d\vec{L} \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$

note: technically, this should be a closed integral – Currents don't start + stop like charges.

Example Loop Current

Find Field $\vec{H}$ at center (origin) of a counter clockwise current loop carrying current I with radius R, lying in the xy plane.



$$\vec{r} = 0\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$$

$$\vec{r}' = R\cos\phi\,\hat{a}_x + R\sin\phi\,\hat{a}_y$$

$$d\vec{L} = \underbrace{-\rho}_{-R} \sin\phi\, d\phi\, \hat{a}_x + \underbrace{\rho}_{+R}\cos\phi\, \hat{a}_y$$

Example Loop Current

$$d\vec{L} = \rho\, d\phi\, \hat{a}_\phi = R\, d\phi\, \hat{a}_\phi$$

$$\hat{a}_\phi = -\sin\phi\, \hat{a}_x + \cos\phi\, \hat{a}_y$$

$$d\vec{L} \times (\vec{r} - \vec{r}\,') = \det \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -R\sin\phi\, d\phi & R\cos\phi\, d\phi & 0 \\ -R\cos\phi & -R\sin\phi & 0 \end{vmatrix}$$

$$= R^2 \hat{a}_z\, d\phi$$

$$\vec{H} = \int \frac{I\, d\vec{L} \times (\vec{r} - \vec{r}\,')}{4\pi \|\vec{r} - \vec{r}\,'\|^3} = \frac{I}{4\pi R} \int_0^{2\pi} d\phi\, \hat{a}_z = \frac{I}{2R} \hat{a}_z$$

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Other Forms of Current

- I – Line current (Amps or C/s)
  - Useful for thin-wire problems
  - Single integral over path
- Ks – Surface current density (A/m or C/s/m)
  - Useful for "skin" currents
  - Double integral over a surface
- Js – Volume current density (A/m^2 or C/s/m^2)
  - Useful for bulk conductivity problems
  - Triple integral over volume

$$\vec{H}(\vec{r}) = \int_L \frac{I d\vec{L} \times (\vec{r} - \vec{r}\,')}{4\pi \|\vec{r} - \vec{r}\,'\|^3}$$

$$= \iint_S \frac{\vec{K}(\vec{r}\,') \times (\vec{r} - \vec{r}\,') dS}{4\pi \|\vec{r} - \vec{r}\,'\|^3}$$

$$= \iiint_V \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,') dV}{4\pi \|\vec{r} - \vec{r}\,'\|^3}$$

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