MSF3: Ampere's Law

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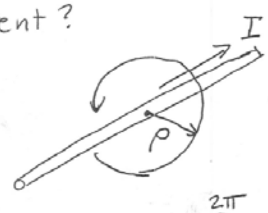
Ampere's Law in Integral Form

$$\oint \vec{H} \cdot d\vec{L} = I$$

How does this work on the line of current?



$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

$$\oint \vec{H} \cdot \underbrace{d\vec{L}}_{\rho d\phi \hat{\phi}} = \int_0^{2\pi} \frac{I}{2\pi} d\phi = I$$

Total Magnetic field around a path is equal to the enclosed current

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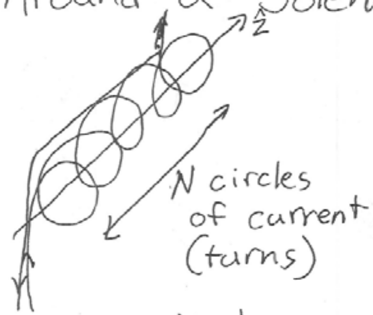
Ampere's Law Applied to a Solenoid

Around a Solenoid:



approximate as a cylinder of Sheet Current

Any path around the Solenoid should produce 0

$$\oint_{\text{outside}} \vec{H} \cdot d\vec{L} \cong 0$$

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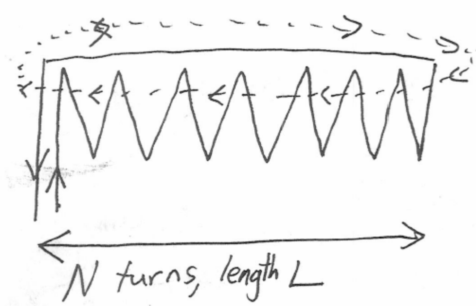
Path Integral Through Solenoid

But what about an interior path



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Estimate of Magnetic Field Inside Solenoid

$$\oint \vec{H}\cdot d\vec{L} = \int_{\text{outside}} \cancel{\vec{H}_{out}\cdot d\vec{L}}^{\,0} + \int_{\text{inside}} \vec{H}_{in}\cdot d\vec{L} = NI$$

$$\vec{H}_{in} \cong H_o\,\hat{a}_z \qquad d\vec{L} = dz\,\hat{a}_z$$

$$\int_0^L H_o\,dz = NI$$

$$H_o = \frac{NI}{L}, \qquad \vec{H} = \frac{NI}{L}\,\hat{a}_z$$

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Derivation of the Point-Form of Ampere's Law

Integral Amperes Law

$$\oint \vec{H}\cdot d\vec{L} = I = \iint_A \vec{J}\cdot d\hat{n}$$

Line Integral of $\vec{H}$-field = Total Current through the enclosed area

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Stoke's Theorem for a Vector Field

$$\oint \vec{A}(\vec{r}) \cdot d\vec{L} = \iint_A \nabla \times \vec{A}(\vec{r}) \cdot d\hat{n}$$

$$\nabla \times \vec{A} = \det \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \qquad \vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

$$\nabla \times \vec{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{a}_x + \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\hat{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{a}_z$$

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Stoke's Theorem Applied to EM Fields

Thus we can write (from a purely Mathematical Relationship)

$$\oint \vec{H} \cdot d\vec{L} = \iint_A \nabla \times \vec{H} \cdot d\hat{n} = \overbrace{\iint_A \vec{J} \cdot d\hat{n}}^{I}$$

and Even

$$\oint \vec{E} \cdot d\vec{L} = \iint_A \nabla \times \vec{E} \cdot d\hat{n} = 0$$

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Summary of Point Form of Field Equations

Point form of Maxwell's Equations

$$\nabla \times \vec{H} = \vec{J} + \boxed{\quad} \qquad \boxed{\qquad\qquad\qquad}$$

$$\nabla \times \vec{E} = \vec{0} + \boxed{\quad} \qquad \nabla \cdot \vec{D} = \rho_v$$

The curl of a static electric field is always $\vec{0}$.