MSF5: Faraday's Law

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Transition to Electrodynamics

This is the first electrodynamic law we've studied

We know that areas in space with magnetic fields have magnetic flux given by surface-normal integral:

$$\Phi_m = \iint_S \vec{B} \cdot d\hat{n}$$

Inference of Faraday's Law from Circuit Theory

Consider the capacitor

$$L = \frac{\Phi_m}{I} \qquad \Phi_m = LI$$

We are used to $V = L\frac{dI}{dt}$

$$\frac{d\Phi_m}{dt} = L\frac{dI}{dt} \rightarrow V = \frac{d\Phi_m}{dt}$$

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Integral Form of Faraday's Law

$$\frac{d\Phi_m}{dt} = L\frac{dI}{dt} \rightarrow V = \frac{d\Phi_m}{dt}$$

$V = -\oint \vec{E}\cdot d\vec{L}$ closed Loop Voltage

Faraday's Law:

$$\oint \vec{E}\cdot d\vec{L} = -\frac{d}{dt}\oiint \vec{B}\cdot d\hat{n}$$

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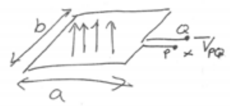
Example: Time-Varying Magnetic Field

Time Varying $\vec{B}$ in a Rectangular Circuit.

$$\vec{B} = B_o \exp(-\alpha t)\hat{a}_z$$

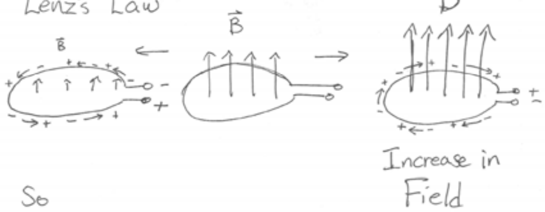


$$\Phi_m = a b B_o \exp(-\alpha t)$$

$$V_{emf}(t) = -\frac{d\Phi_m}{dt} \qquad V_{PQ}$$

why − ?

Lenz's Law



So

$$V_{emf} = + a b B_o \alpha \exp(-\alpha t)$$

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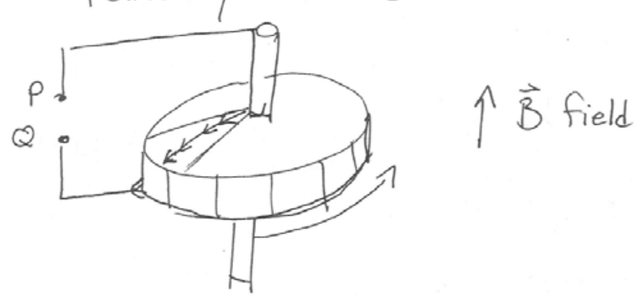
Faraday Disk Generator

Faraday Disk Generator



$$V_{emf} = \frac{1}{2} B \rho_o^2 \omega$$

ρ_o: radius of wheel; ω: radians/second

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