Chapter 4

MAGNETIC FLUX IN RFID SYSTEMS

This chapter presents an overview of the physical modeling efforts of inductive RFID tags in the context of corrosivity sensing. Tutorial in nature, the variety of behaviors arising from magnetic flux coupling from reader to RF tag are explored. Specific to the corrosivity application, we explore how operation of an inductive RFID tag in free space differs from its operation on metal or an RF isolator pad. Also discussed are the effects of parasitics and reader antenna dimensions and orientation. Overall, this chapter provides the background required to interpret many of the simulation results in Chapter **??**.

4.1 Magnetic Field Modeling

This section presents the theoretical models and mechanisms that describe the operation of an inductive RFID system near metal surfaces and parasitics.

4.1.1 Free Space Operation

In free space, the operation of a square-loop current reader operates efficiently, producing a swirl of oscillating magnetic flux through and around its aperture. This flux is illustrated in Figure 4.1. The flux lines were calculated using a basic Biot-Savart integration of the square current elements. In this and subsequent analysis, we employ a *quasi-static* assumption that allows us to equate the magnitudes of magnetostatic fields (those due to DC currents) to the 13.56 MHz fields in the inductive RFID system. This assumption is valid because the overall dimensions of our calculation are much less than a free-space wavelength (22 meters at 13.56 MHz).

Note that there are three typical read configurations for this RFID system: axial, transverse, and lateral. Each configuration – depicted in Figure 4.1 – has its benefits and drawbacks and *all* configurations experience severe loss of power-coupling with the square coil as the separation distances increase. For the corrosivity sensor, the lateral configuration is the least useful as it does not couple sufficient power when the tag is placed on metal.

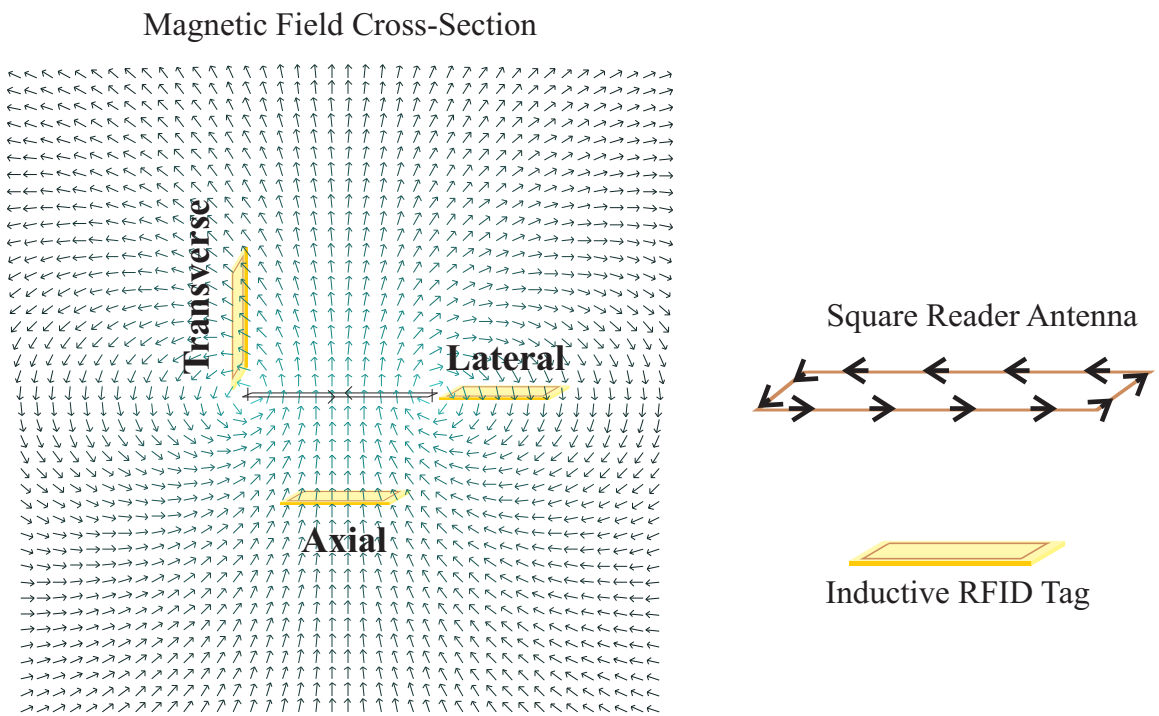
4.1.2 Axial Operation on a Perfect Conductor

When a perfect electric conductor (PEC) is introduced to problem, there will be a dramatic reduction in magnetic field amplitudes close-in to the surface. Field strength calculation in this situation is accomplished using *image theory*. Image theory states that the total field above a PEC is calculated by adding the free-space radiated fields of the square loop with its perfect image, reflected about the flat surface of the PEC. This image has equal and opposite current flows which cancel all normal-components of magnetic field on the surface of the conductor. This behavior is illustrated in Figure 4.2 for the axial configuration.

The presence of the metal is debilitating to RFID tag coupling because there are no longer normal magnetic fields contributing to the total flux through the coil. Without normal flux, Faraday's law predicts no voltage excitation around the coil. Only the marginal thickness of the dielectric substrate will allow a little magnetic flux through the tag.

It is important to note that there are no real currents deep within the PEC medium; all of the cancelation currents are excited on the *surface* of the PEC. They simply mimic the equivalent behavior of a mirror-image source beneath the conductive medium. If the metal surface has finite conductivity, then the surface currents imperfectly mimic this mirror source. To a rough approximation, we may offset the image current further from the surface by a distance equal to the skin

Figure 4.1. Sketch of magnetic flux around a square loop operating in free space.



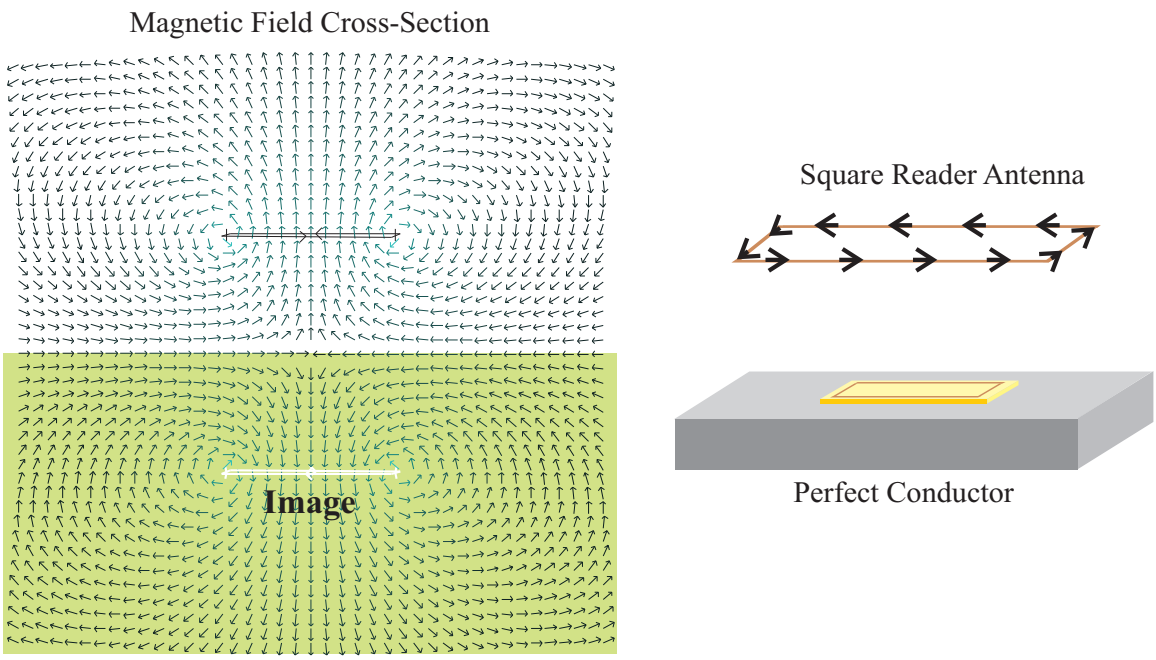
depth:

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \tag{4.1.1}$$

where f is frequency (in Hz), μ is permeability (in H/m), and σ is conductivity (in S/m).

Based on this model, when the metal is a PEC, $\sigma \to \infty$ S/m and the skin depth approaches zero; the image currents exactly reflect across the flat surface. When the metal becomes a dielectric material with $\sigma \to 0$ S/m, the skin depth goes to infinity and the image is pushed so far away as to leave the surface fields identical to free space. Finite conductivities represent in-between cases where the mirror current is pushed downward by a skin-depth (which can be shown to be the centroid depth of the surface currents).

Figure 4.2. Sketch of magnetic flux around an axial-configuration, square-loop reader operating in the presence of a perfectly conducting metal surface.



4.1.3 Transverse Operation on a Perfect Conductor

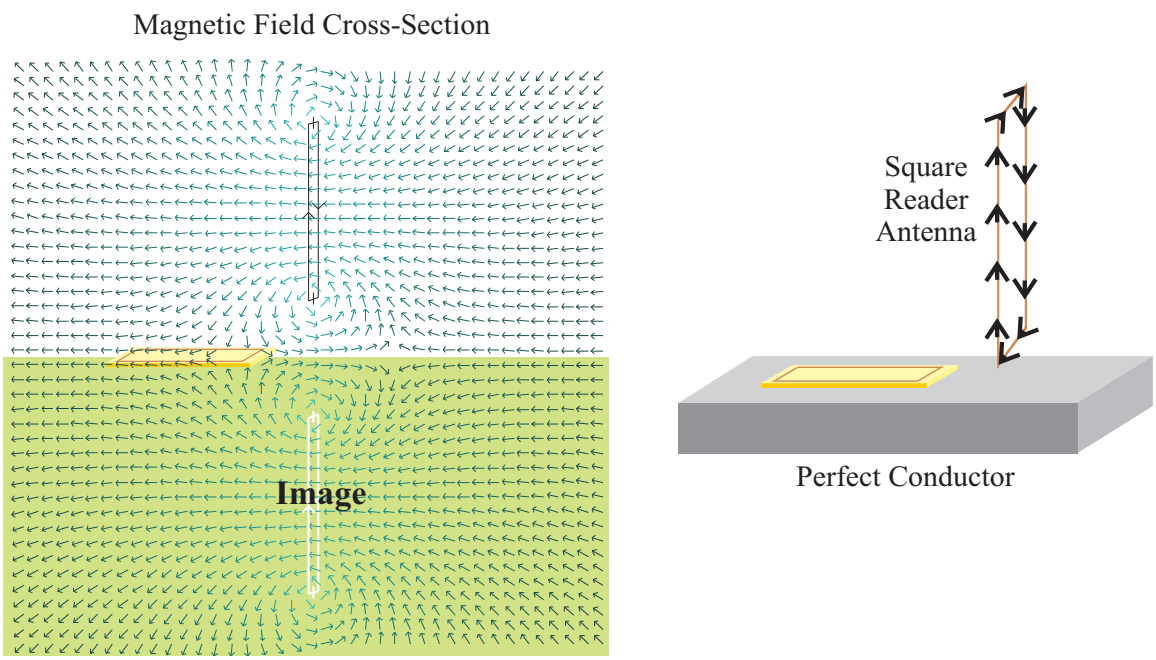
A similar image theory result can be obtained for the transverse reader configuration shown in Figure 4.3. Just as in the axial mode configuration, the image below the PEC surface cancels all normal magnetic fields. Again, on-metal RFID tag operation will be nearly impossible.

Note also in Figure 4.3 that there is a particularly strong null spot just to the right and to the left of the transverse reader loop. In fact, the density of magnetic flux across the RFID tag varies more for the transverse excitation than the case of axial excitation. If care is not taken to consistently align the square reader antenna during the measurement, the transverse measurement may prove to be less repeatable than the axial measurement.

4.2 RF Tag Isolation

A key method for mitigating the on-metal degradations of the RFID tag coupling is to place the tag on an RF isolator. There are several types of isolator, the

Figure 4.3. Sketch of magnetic flux around a transverse-configuration, square-loop reader operating in the presence of a perfectly conducting metal surface.



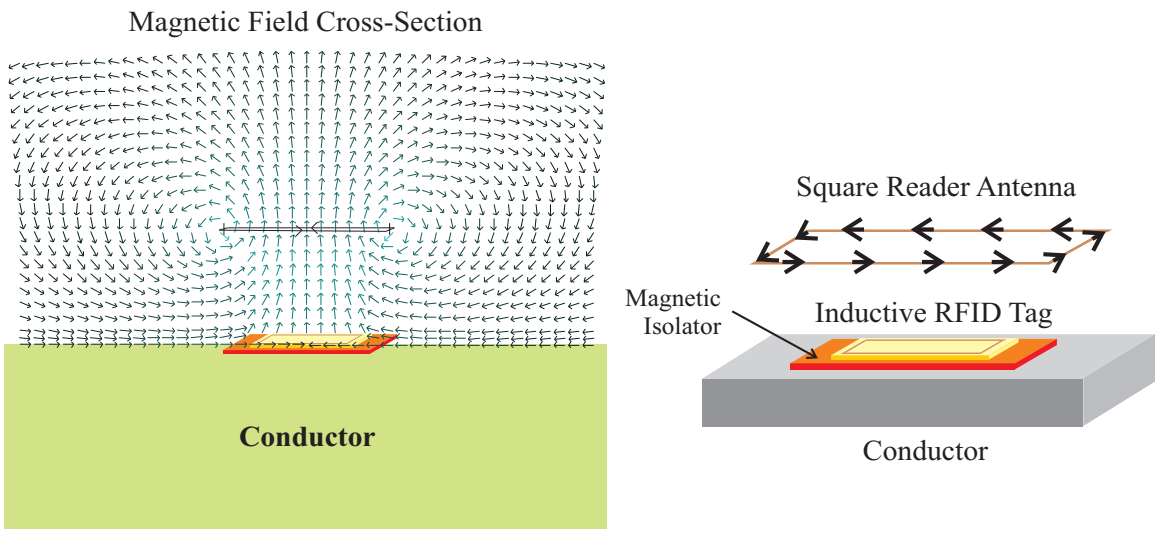
most useful at 13.56 MHz being a magnetic, non-conducting pad. This pad is made from polymers with ferrite particles sprinkled throughout the medium. This construction allows a thin, flexible pad that has a significant permeability with very low conductivity. Thus, magnetic flux is drawn into the pad, which will not contain eddy currents to cancel the field. The flux lines will then leave the pad through the thin edges.

Figure 4.4 illustrates the flux through the RFID tag with magnetic isolator for the axial configuration. Note that normal magnetic flux is allowable at the surface of the isolator pad. Magnetic flux must be conserved, however, so the tangential field lines increase along the edge of the isolator pad to allow the flux a path for leaving the RFID tag.

Figure 4.5 illustrates the same isolator pad effects used in the transverse reader configuration. Again the magnetic pad draws more flux through the RFID tag coil and recovers much of the power lost by the presence of the nearby metal surface. At frequencies much higher than 13.56 MHz, conventional magnetic materials begin to fail. Thus, the RF pads used at 13.56 MHz would not necessarily provide the same

protection at UHF or microwave frequencies.

Figure 4.4. Sketch of magnetic flux around an axial-configuration, square-loop reader operating near a perfect conducting metal surface with an RFID tag on a magnetic RF isolator pad.



4.3 Magnetic Flux Circuit Analysis

Aside from electrical circuits, there are actually many linear systems that can be modeled with basic circuit theory. One such system is the flow of magnetic flux through inhomogeneous materials. In this system, magnetic flux takes the place of electrical current in a conventional circuit; instead of voltage sources, the magnetic circuit is excited by *magneto-motive force* – a loop or coil of current that effectively energizes the magnetic flux.

Because magnetic flux is neither created nor destroyed, it follows a Kirchhoff current law just like electrical current. The net magnetic flux into any node within the circuit must be zero. Likewise, magneto-motive force satisfies the same conservation properties of its counterpart, voltage, in electric circuits. When summed around any arbitrary loop, the quantity we define as total magneto-motive force must equal zero in the magnetic circuit – just like Kirchhoff's voltage law.

To complete the analogy, we need a physical quantity in a magnetic circuit to serve as an analogy to resistance. Then we can apply Ohm's law and calculate how magnetic flux might distribute itself in an inhomogeneous collection of materials. We will call this term *reluctance*, $\mathcal{R}$, which will quantify how easily magnetic flux

Chapter 5

INDUCTIVE FIELD MODELING

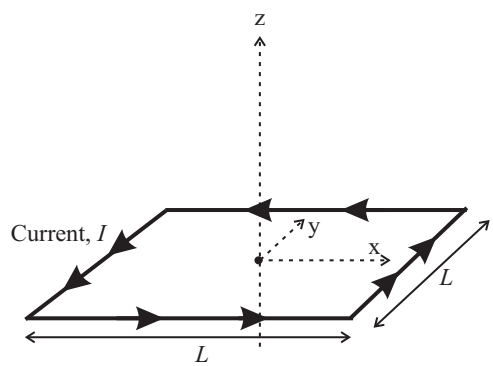
If you are a technical reader that has made it this far in the text, there is no doubt that you qualitatively understand the basic principle of power and information coupling in an inductive RFID system. It is another thing altogether to possess the mental tools for quantitative analysis of these systems. Developing those tools is the primary goal of this chapter.

5.1 Magnetic Field Modeling

5.1.1 Magnetic Field Around a Current Loop

We will model the excitation at the reader antenna as a square loop of current with side lengths L. This square will be centered at the origin and parallel with the xy-plane, as illustrated in Figure 5.1. Given a single line current I, we will use the Biot-Savart relationship to calculate the $\vec{\mathrm{H}}$-field along the z-axis ($\vec{\mathrm{H}}(0,0,z)$). Off-axis behavior of the field is more difficult to calculate, but also unnecessary if we assume that the RFID tag is roughly in the center of the reader antenna's field-of-view.

Figure 5.1. Magnetic flux flows through the square coil as a function of current and point of observation.



Here follows a step-by-step field analysis of the square current loop in Figure 5.1.

1. The Biot-Savart relationship states that the total magnetic field due to a current in space is given by the following path integrations:

$$\vec{H}(\vec{r}) = \oint_L \frac{I d\hat{l} \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3} \tag{5.1.1}$$

where $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ is the observation point, $\vec{r}' = x'\hat{x} + y'\hat{y} + z'\hat{z}$ marks the variables of integration, and I is the current in Amps. The integral in Equation (5.1.1) is taken around the closed current path. The first step is to pick a differential element of integration:

$$\text{Current in } x\text{-direction: } d\hat{l} = dx'\hat{x}$$

$$\text{Current in } y\text{-direction: } d\hat{l} = dy'\hat{y}$$

This problem actually consists of 4 different current segments that travel in two different directions. Two travel along x and two travel along y.

2. Next, pick the limits of integration:

$$\oint_L \frac{Id\hat{l} \times (\vec{r} - \vec{r}')}{4\pi\|\vec{r} - \vec{r}'\|^3} = \underbrace{\int_{-L/2}^{L/2} \frac{Idx'\,\hat{x} \times (z\hat{z} - x'\hat{x} + \frac{L}{2}\hat{y})}{4\pi\|z\hat{z} - x'\hat{x} + \frac{L}{2}\hat{y}\|^3}}_{\text{Segment 1}} + \underbrace{\int_{-L/2}^{L/2} \frac{Idy'\,\hat{y} \times (z\hat{z} - \frac{L}{2}\hat{x} - y'\hat{y})}{4\pi\|z\hat{z} - \frac{L}{2}\hat{x} - y'\hat{y}\|^3}}_{\text{Segment 2}} +$$

$$\underbrace{\int_{L/2}^{-L/2} \frac{Idx'\,\hat{x} \times (z\hat{z} - x'\hat{x} - \frac{L}{2}\hat{y})}{4\pi\|z\hat{z} - x'\hat{x} - \frac{L}{2}\hat{y}\|^3}}_{\text{Segment 3}} + \underbrace{\int_{L/2}^{-L/2} \frac{Idy'\,\hat{y} \times (z\hat{z} + \frac{L}{2}\hat{x} - y'\hat{y})}{4\pi\|z\hat{z} + \frac{L}{2}\hat{x} - y'\hat{y}\|^3}}_{\text{Segment 4}}$$

For this problem, our integral breaks into four pieces.

3. For observation on the z-axis, we will apply symmetry arguments. If all 4 current segments are equal, then there should be no x or y components of field along the z axis. The two x-aligned segments will produce equal and opposite magnetic fields in the y-direction and the two y-aligned segments will produce equal and opposite magnetic fields in the x-direction. All four, however, will contribute equal amounts in the z-direction. Thus, we could write:

$$\vec{H}(0,0,z) = H_z(z)\hat{z} \qquad H_z(z) = 4\hat{z} \cdot \underbrace{\int_{-L/2}^{L/2} \frac{Idx'\,\hat{x} \times (z\hat{z} - x'\hat{x} + \frac{L}{2}\hat{y})}{4\pi\|z\hat{z} - x'\hat{x} + \frac{L}{2}\hat{y}\|^3}}_{\text{Segment 1}}$$

4. Simplify the integral:

$$\begin{aligned} H_z &= 4\hat{z} \cdot \underbrace{\int_{-L/2}^{L/2} \frac{Idx'\,\hat{x} \times (z\hat{z} - x'\hat{x} + \frac{L}{2}\hat{y})}{4\pi\|z\hat{z} - x'\hat{x} + \frac{L}{2}\hat{y}\|^3}}_{\text{Segment 1}} \\ &= \frac{IL}{2\pi} \int_{-L/2}^{L/2} \frac{dx'}{\left(z^2 + \frac{L^2}{4} + x'^2\right)^{\frac{3}{2}}} \end{aligned}$$

$$= \frac{IL}{2\pi\left(z^2+\frac{L^2}{4}\right)} \frac{x'}{\sqrt{z^2+\frac{L^2}{4}+x'^2}}\Bigg|_{x'=-\frac{L}{2}}^{x'=\frac{L}{2}}$$

$$= \frac{IL}{2\pi\left(z^2+\frac{L^2}{4}\right)} \frac{L}{\sqrt{z^2+\frac{L^2}{2}}}$$

After all the simplifications, the final answer is

$$\vec{H}(0,0,z) = \frac{I}{2\pi\left(\frac{z^2}{L^2}+\frac{1}{4}\right)\sqrt{z^2+\frac{L^2}{2}}}\hat{z} \tag{5.1.2}$$

Equation (5.1.2) is a key result the illustrates why the range of inductive RFID is so limited. For close-in operation ($z \ll L$), the magnetic field becomes independent of z:

$$\vec{H}(0,0,z) \approx \frac{2\sqrt{2}I}{\pi L}\hat{z} \tag{5.1.3}$$

When the RFID tag is distant from the reader antenna (further than a side length L, such that $z \gg L$), the magnetic field falls off rapidly:

$$\vec{H}(0,0,z) \approx \frac{IL^2}{2\pi z^3}\hat{z} \tag{5.1.4}$$

The magnetic field – and mutual inductance – fall off as a function of $1/z^3$. Since the ability of the RFIC to reflect power back through the system varies with the *square* of mutual inductance, extra distance has a truly crippling effect on the power coupling.

5.1.2 Field Strength vs. Distance

The current in the reader coil oscillates at $f = 13.56$ MHz and, following Faraday's Law, excites a voltage around the coil in the RFID tag. This voltage is then rectified by the chip to provide power to the memory and communication circuitry. Power available for coupling into an inductive RFID will be proportional to the magnitude-*squared* of the magnetic field, $\vec{H}$. With N_{ant} turns in the reader antenna, we may adapt Equation (5.1.2) for use along the z-axis:

$$\vec{H}(0,0,z) = \frac{N_{\text{ant}}I}{2\pi\left(\frac{z^2}{L^2}+\frac{1}{4}\right)\sqrt{z^2+\frac{L^2}{2}}}\hat{z}$$

Plotting the normalized power present in the magnetic field, $\|\vec{H}(0,0,z)\|^2/\|\vec{H}(0,0,0)\|^2$, produces the graph in Figure 5.3. Notice that when the card moves more than L away from the reader coil, the energy density in the static magnetic field is reduced

Figure 5.2. To-scale image of a common 13.56 MHz RFIC card with insert removed.

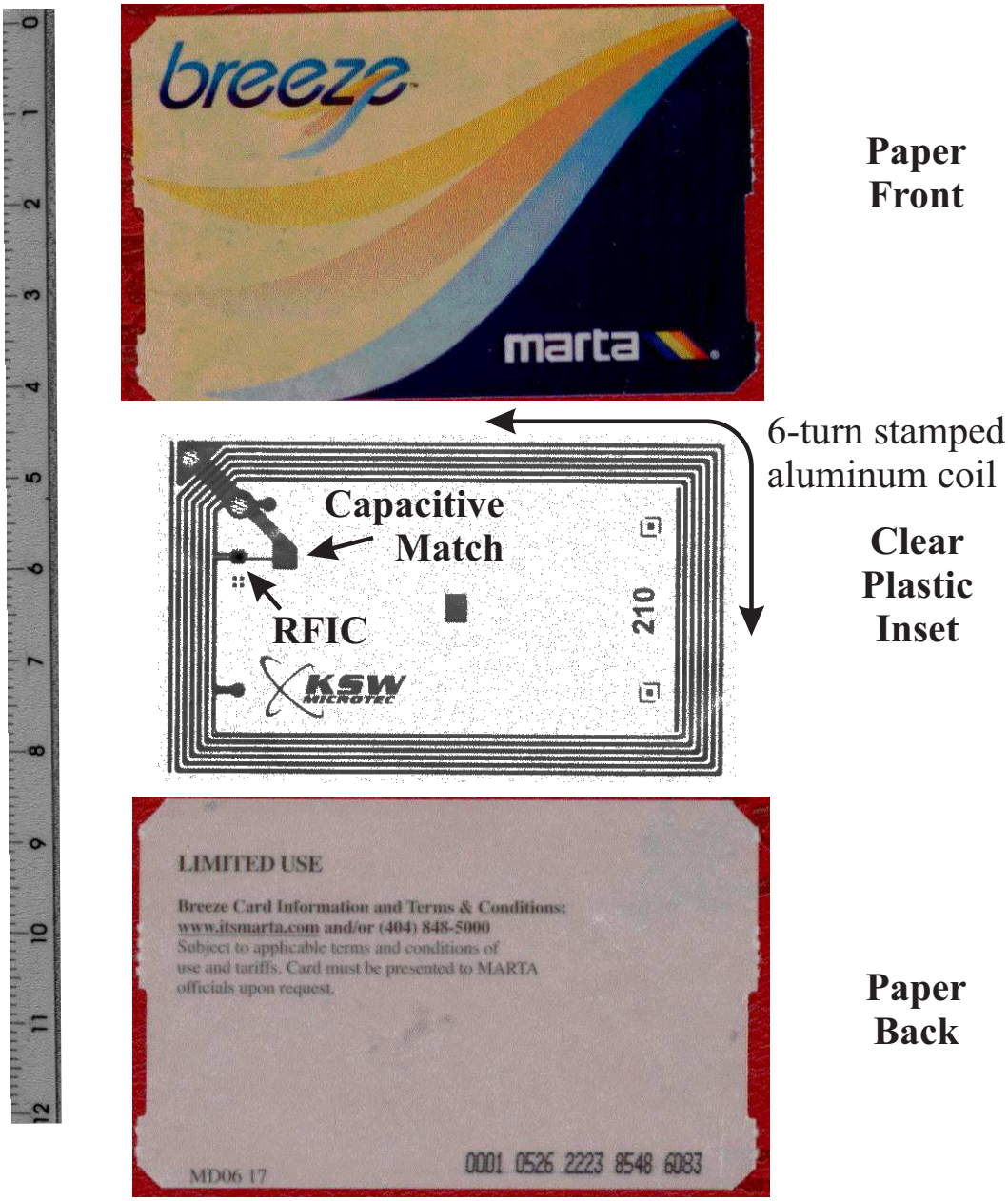
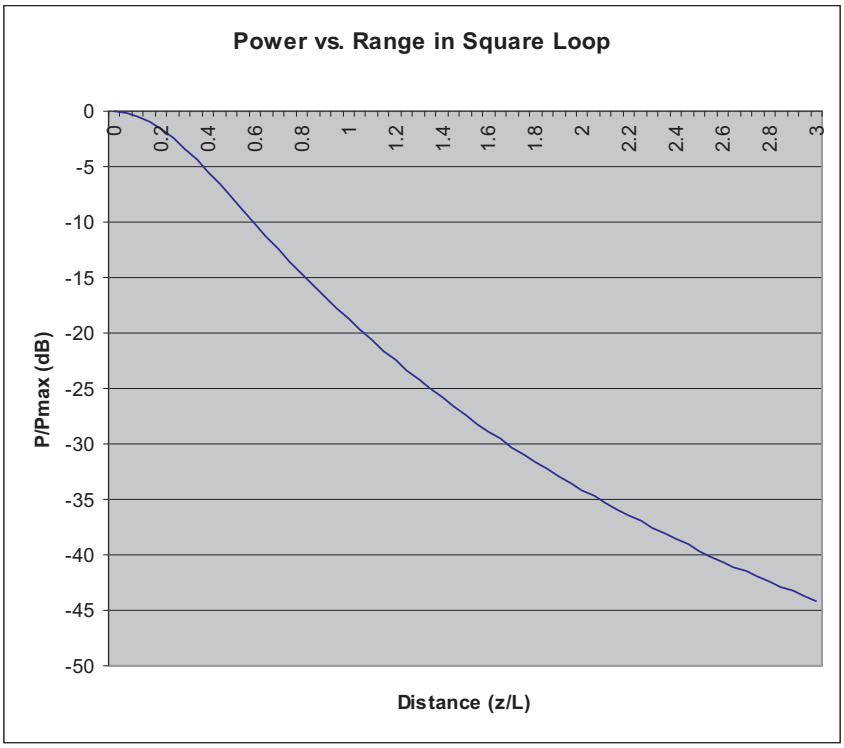


Figure 5.3. Graph of normalized power in the magnetic field for increasing tag-reader separation distance.



by a factor of 100! Since most readers cease free-space operation at approximately a distance L, me may assume that RFID systems can tolerate 20 dB of field-strength loss compared to the ideal case (free-space operation with the RFID tag placed in the center of the reader coil).

5.2 Inductance and Magnetic Coupling

5.2.1 Self Inductance

5.2.2 Mutual Inductance

Now we will estimate the mutual inductance between an RFID tag centered on the z-axis, parallel to the reader coil, and z units away from the plane of the coil. We will approximate the magnetic field as constant across the area of the RFID tag,

since the tag is relatively small compared to the reader antenna. In free space along the z-axis, we may write the magnetic flux density as

$$\vec{B}(0,0,z) = \frac{N_{\text{ant}} I \mu_0}{2\pi\left(\frac{z^2}{L^2}+\frac{1}{4}\right)\sqrt{z^2+\frac{L^2}{2}}}\hat{z}$$

Mutual inductance is defined as the ratio of total flux through both coils and the current through the coils at the reader, $M = \Psi_{21}/I$.

For a tag with N_{tag} turns, the total magnetic flux is approximately:

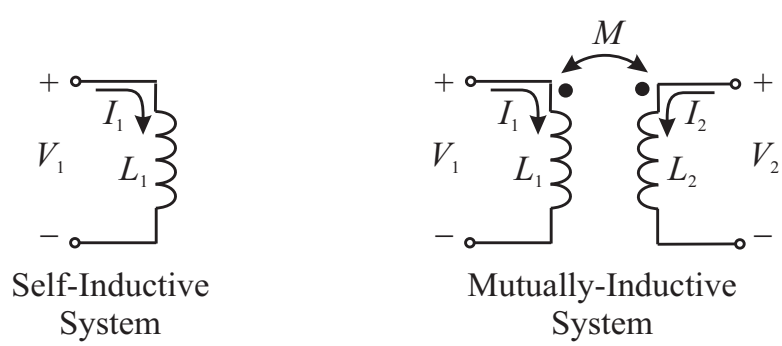
$$\Psi_{21} = N_c A \|\vec{B}(0,0,z)\| = \frac{N_{\text{ant}} N_{\text{tag}} A_{\text{tag}} I \mu_0}{2\pi\left(\frac{z^2}{L^2}+\frac{1}{4}\right)\sqrt{z^2+\frac{L^2}{2}}}$$

where A_{tag} is the card area (approximately 0.0015 m^2). Thus, mutual inductance in this system is

$$M = \frac{\Psi_{21}}{I} = \frac{N_{\text{ant}} N_{\text{tag}} A_{\text{tag}} \mu_0}{2\pi\left(\frac{z^2}{L^2}+\frac{1}{4}\right)\sqrt{z^2+\frac{L^2}{2}}}$$

This allows us to construct Thevenin equivalent circuit models for the entire free space system.

Figure 5.4. Circuit models for (left) self-inductive systems and (right) mutually-inductive systems.



5.2.3 Mutual Inductance Circuit Modeling

To create a system of equations that describes the mutually inductive system in Figure 5.4, we have to first recognize that there is an interdependence between currents and voltages that do not exist in simpler circuit components. Namely, the currents flowing as I_1 and I_2 in the circuit of Figure 5.4 will *both* influence the

terminal voltages of port 1 and port 2. Working from the first-principles circuit modeling time domain equations, we may write

$$V_1 = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} \qquad V_2 = L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt} \tag{5.2.1}$$

which would look like two stand-alone inductors if the second terms were removed (i.e. mutual inductance M vanished). When present, however, the second mutual inductance allows current I_2 to excite additional voltage on port 1 and current I_1 to excite additional voltage on port 2 consistent with Faraday's law of induction. The sign is negative in the inductive term because the flux *leaving* one set of coils enters into the second set of coils with opposite polarity of self-inductive fields.

There is a much more elegant way to write the interdependent set of relationships in Equation (5.2.1) using matrix notation. For a given frequency f, we may write the matrix relationship between phasor voltages and currents as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j2\pi f \begin{bmatrix} L_1 & -M \\ -M & L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{5.2.2}$$

which is a simple linear system of algebraic equations. From this relationship, we can construct a way to estimate how a load connected to port 2 influences the equivalent impedance seen at port 1 of the mutually inductive system.

If the equivalent resistance at port 1 of Figure 5.4 is given by $Z_1 = V_1/I_1$, while the impedance connected to port 2 is given by $Z_2 = V_2/I_2$, then these two impedances are related by the following equation:

$$Z_1 = \frac{V_1}{I_1} = j2\pi f L_1 + \frac{4\pi^2 f^2 M^2}{Z_2 + j2\pi f L_2} \tag{5.2.3}$$

This relationship illustrates how a load of Z_2 will reflect through the system and influence the Thevenin equivalent impedance of the system Z_1. Notice that there are two terms in Equation (5.2.3). The first term is a large self-inductive term that typically dominates the total impedance Z_1. Because typical mutual inductance is much smaller than the self-inductance terms ($M^2 \ll L_1 L_2$), the right-hand term that contains Z_1 is much smaller – yet this is where the information exchange occurs. The next section illustrates how this circuit model may be used to analyze inductive RFID systems.

5.3 Equivalent Circuit Model for the Basic RFID Card System

This section describes a first-principles equivalent circuit model for the RFID system that includes

5.3.1 Circuit Components of an RFID System

To construct an equivalent circuit model of an inductive RFID system, we need to add some complexity to the simple mutual inductive system of Figure 5.4. To be

realistic, the model will need to incorporate the following physical attributes of an RF tag system:

L_{ant} **Antenna Self-Inductance:** The antenna loop used to excite the RFID system will have a self-inductance term regardless of whether nearby RFID tags are coupling into its wire currents.

R_{ant} **Antenna Loop Resistance:** Any realistic antenna loop will have non-zero resistance around its current path. The engineer always attempts to minimize this term since it represents Ohmic losses in the system.

V_s **RF Reader Voltage:** This is the magnitude of the voltage source of the RFID reader, which may also be represented as a current source.

R_s **RF Reader Resistance:** This is the source resistance of the reader.

C_{read} **Antenna Matching Capacitance:** This capacitance, which is usually tunable, helps to counter the antenna self-inductance and allows maximum power transfer to and from coupled RF tags.

M **Mutual Inductance:** This is the mutual inductance between the RFID tag and the reader antenna, which depends on how much magnetic flux is shared between the two. This is a function of orientation, tag-reader separation distance/position, tag coil turns and geometry, antenna loop turns and geometry, and material environment.

L_{tag} **Tag Self-Inductance:** This term is the self-inductance of the RFID tag coil, independent of what reader may be coupling to the device.

R_{coil} **Tag Coil Resistance:** The total resistance in the RFID tag coil represents Ohmic losses along the tag's main current path. The RFID tag always functions better when this term is minimized.

Z_{RFIC} **RFIC chip impedance:** This is the Thevenin impedance of the RF integrated circuit connected to the RFID tag coil. Note that this value may change for a single RFIC, depending on whether the chip is powering-up, absorbing power in the steady-state, or modulating information back to the reader.

C_{ext} **Tag Matching Capacitance:** This external matching capacitance is used to match the tag's RFIC with the inductive tag coil. If chosen correctly, this matching capacitance will maximize the influence of the RFIC's impedance changes on the terminal impedance of the reader antenna.

Figure 5.5 illustrates the connectivity of these physical quantities. Armed with this circuit model, it will be possible to illustrate how an RFID system quantitatively operates. It will also be able to highlight critical design features of such a system.

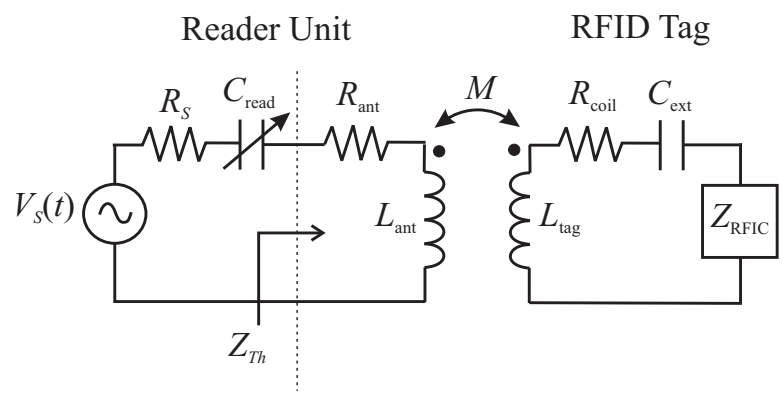


Figure 5.5. Equivalent circuit model for an inductive RFID reader in the presence of a card.

5.3.2 Example RFID Tag System

A custom circuit model for an inductive RFID system, illustrated in Figure 5.5, was designed by Georgia Tech researchers. The physics-based circuit models the system as a lossy transformer, with mutual inductance M calculated from a series of geometrical parameters including coil turns, tag-reader separation distance, tag size, and reader antenna size. The Thevenin equivalent impedance for two mutually-coupled circuits is

$$Z_{Th} = R_{\text{ant}} + j2\pi f L_{\text{ant}} + \frac{4\pi^2 f^2 M^2}{Z_{\text{RFIC}} + R_{\text{coil}} + j\left(2\pi f L_{\text{tag}} - \frac{1}{2\pi f C_{\text{ext}}}\right)}$$

With this expression, it becomes possible to estimate power coupling between reader and the tag's *radiofrequency integrated circuit* (RFIC).

Table 5.1. Table of typical circuit component values in a typical RFID system.

Var.	Quantity	Value	Units
f	Frequency	13.56	MHz
L_{ant}	Antenna Loop Inductance	40.0	μH
C_{read}	Antenna Tuning Capacitance	X	pF
R_{ant}	Antenna Loop Resistance	X	Ω
L_{tag}	Tag Coil Inductance	7.8	μH
C_{ext}	Tag Matching Capacitance	24	pF
R_{coil}	Tag Coil Resistance	70	Ω
Z_{RFIC}	RFIC Impedance	X	Ω