# **Curriculum Topic** :
# Time-Domain Transmission Lines

## **TDT2** : Transmission Line Equations

| *Module Outline:* | |
|---|---|
| Prerequisite Skills | Competencies |
| Supplemental Reading and Resources | Assessments |
| Laboratory Activities | Power Point Slides and Notes |

## Prerequisite Skills

*Prerequisites / Requirements:*
**TDT1** Introduction to Transmission Lines

## Competencies

**Competency TDT.2:** **Link the physical attributes of a transmission line to the properties of electrical signals that they transport**

*Competency Builders:*

| | |
|---|---|
| TDT.2.1 | Recognize the telegrapher's equations |
| TDT.2.2 | Understand the relationship between a transmission line's per-unit-length capacitance and inductance, velocity of propagation, and intrinsic impedance |
| TDT.2.3 | Identify several basic and common topologies for transmission lines |
| TDT.2.4 | Calculate electrical parameters from the geometry and material properties of common transmission line types |

## Supplemental Reading and Resources

*Supplemental Reading Materials:*

> A.F. Peterson and G.D. Durgin. *Transient Signals on Transmission Lines: An Introduction to the Non-Ideal Effects and Signal Integrity Issues in Electrical Systems*. Morgan & Claypool Publishers, 2009. Chapter 2.

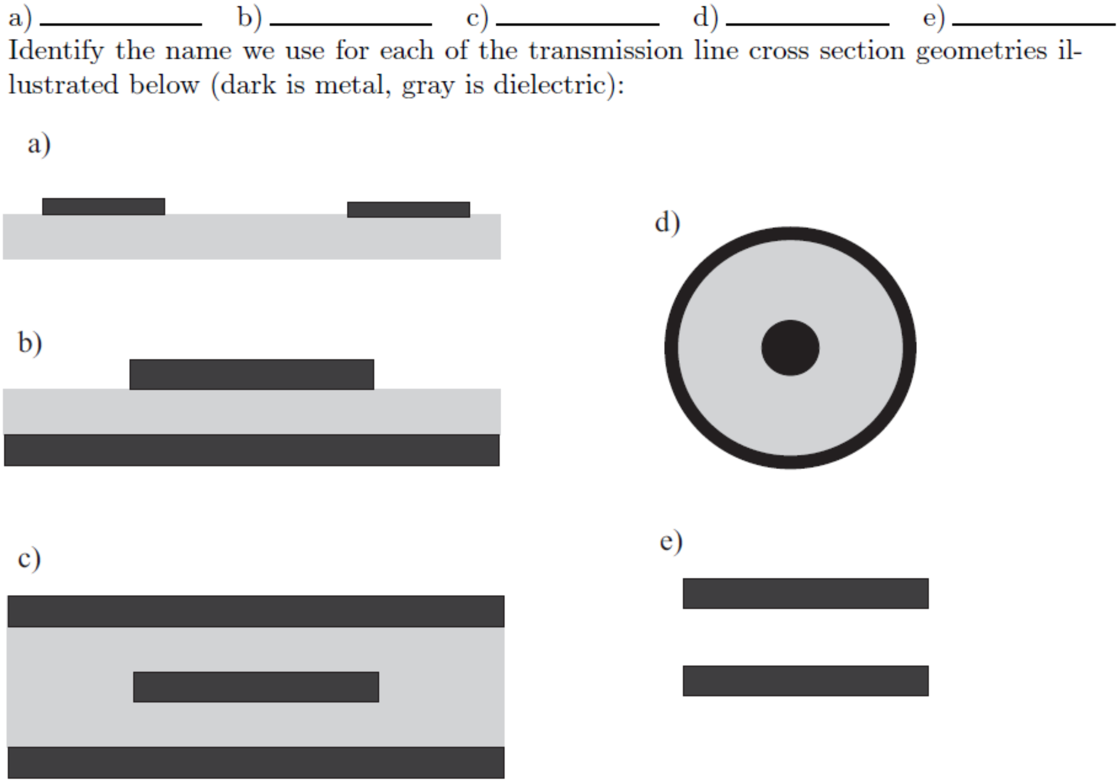
## Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

| *Question:* TDT2.1 | *Competency:* TDT.2.1 |
|---|---|
| The electric __________ was the first application of transmission line theory. | |
| *Answer:* | |
| Telegraph | |

| *Question:* TDT2.2 | *Competency:* TDT.2.2 |
|---|---|
| Signals on a 75Ω transmission line propagate at a velocity of $2 \times 10^8$ m/s. What are the per-unit-length capacitance and inductance of this line? | |
| *Answer:* | |
| 66.7 pF/m and 375 nH/m | |

*Question:* TDT2.3 *Competency:* TDT.2.3



a) ____________ b) ____________ c) ____________ d) ____________ e) ____________

Identify the name we use for each of the transmission line cross section geometries illustrated below (dark is metal, gray is dielectric):

*Answer:*

a) co-planar strip, b) microstrip, c) symmetrical stripline, d) coaxial cable, e) parallel plate

*Question:* TDT2.4 *Competency:* TDT.2.4

You are asked to design a 100 Ω microstrip transmission line that will be etched onto a dielectric substrate with $\varepsilon_r = 3.0$ and a thickness of 4 mm. What should be the width of the microstrip line?

*Answer:*

The easiest way to calculate the width of the microstrip is to use the inversion formula in the notes:

$$a = b\left[\frac{8\exp(A)}{\exp(2A) - 2}\right]$$

where

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}}$$

When these expressions are evaluated for the parameters discussed in the problem statement, the results are $A = 2.49$, $B = 3.42$, and $a = 2.7$ mm.

*Question:* TDT2.5 *Competency:* TDT.2.4

You are asked to design a coaxial cable with $50\Omega$ impedance. The cable must have an outer conductor radius of 1 cm and a solid inner copper core of radius $a$. You must choose a dielectric permittivity $\epsilon_r$ and a conductor core radius $a$ that achieves the desired impedance while minimizing the cost-per-meter of the coaxial cable. Copper conductor costs \$5000 per cubic meter and the dielectric plastic costs $\$(150 + 25\epsilon_r)$ per cubic meter (higher permittivity dielectrics are more expensive). In addition to $\epsilon_r$ and $a$, calculate the material cost-per-foot of your optimum coaxial cable as well as the velocity of propagation, the inductance, and the capacitance.

Hint: It is probably easiest to plot cost vs. $\epsilon_r$ and find the optimal design parameters by visual inspection.

*Answer:*

First, from geometry we know that the volumes of copper and dielectric (per unit length of cable) are given by the following expressions:

$$\text{Cu vol/m: } a^2\pi \qquad \text{Dielectric vol/m: } (b^2 - a^2)\pi$$

The value $b$ is fixed at 0.01m. The per-unit cost of each medium is given by:

$$\text{Cu: } 5000a^2\pi \ \$/\text{m} \qquad \text{Dielectric: } (150 + 25\epsilon_r)(b^2 - a^2)\pi \ \$/\text{m}$$

(Note: there would also be some outer conductor coating, but this is a fixed cost since the cable radius is static.)

We also know that $a$ will be a function of the dielectric permittivity $\epsilon_r$. We must invert the coaxial equation from the notes:

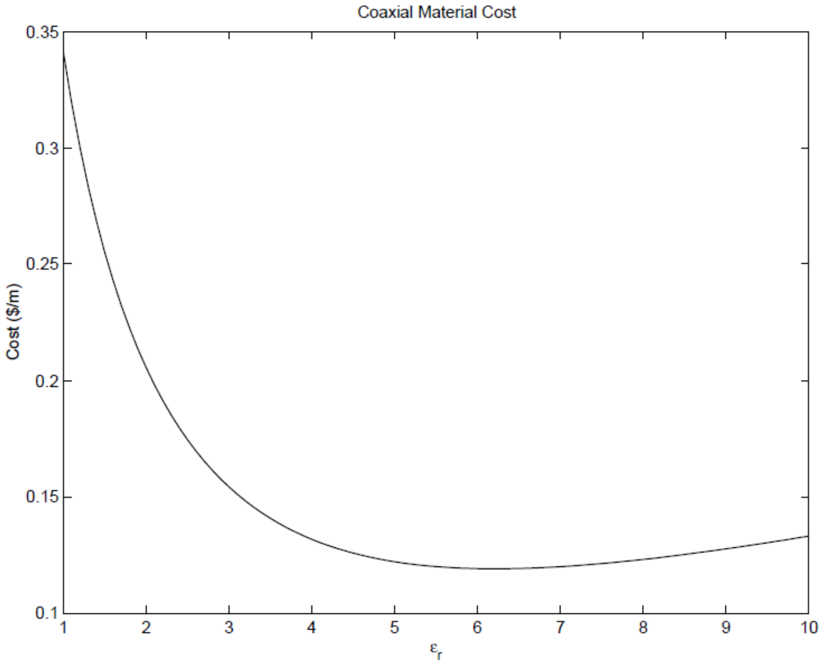
$$Z_0 = \frac{\ln\left(\frac{b}{a}\right)}{2\pi}\sqrt{\frac{\mu}{\epsilon}} \quad \longrightarrow \quad a = b\exp\left(-2\pi Z_0\sqrt{\frac{\epsilon_r\epsilon_0}{\mu_0}}\right)$$

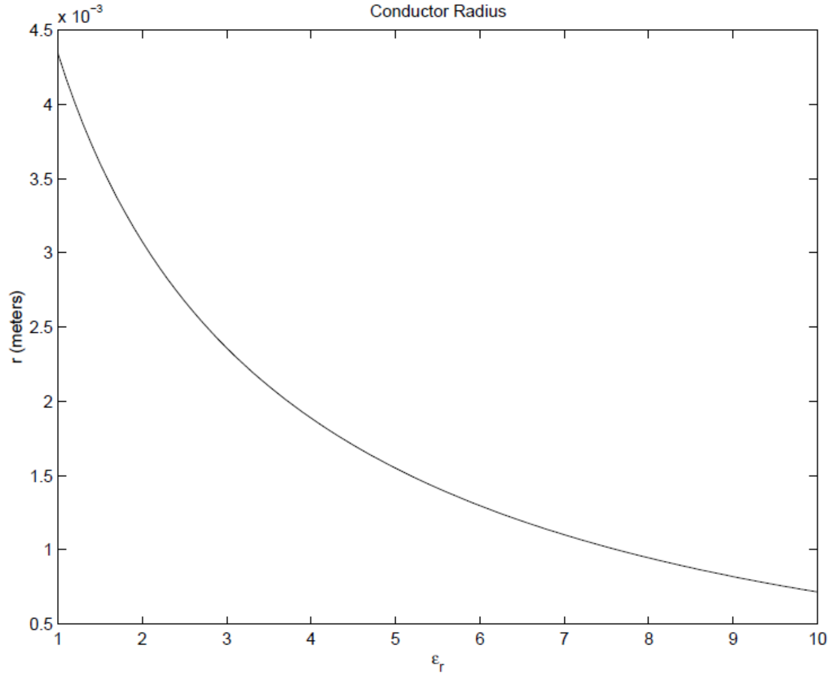
This expression allows us to calculate the inner conductor radius as a function of permittivity (unknown) and target impedance (known). Thus, to compute the total cost of the dielectric and copper core, we can construct the following expressions through substitution:

$$\text{Cu Cost: } 5000\pi\left[0.01\exp\left(-2\pi(50)\sqrt{\frac{\epsilon_r\epsilon_0}{\mu_0}}\right)\right]^2$$

$$\text{Dl Cost: } (150 + 25\epsilon_r)\pi\left((0.01)^2 - \left[0.01\exp\left(-2\pi(50)\sqrt{\frac{\epsilon_r\epsilon_0}{\mu_0}}\right)\right]^2\right)$$

The total cost of the dielectric and copper core is the sum of these two values.
Now let's plot total cost as a function of dielectric parameter $\epsilon_r$ along with the inner core radius:





From visual inspection, we see that the cheapest 50 Ω cable can be made with $\epsilon_r \approx 6.2$ and $a \approx 1.3$ mm at a total cost of 11.9 cents/m. The velocity of propagation would be $1.2 \times 10^8$ m/s ($v_p = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_0}}$). The per-unit capacitance and inductance are $1.67 \times 10^{-10}$ F/m and $4.16 \times 10^{-7}$ H/m, respectively.

The Matlab code used to make the graphs for this solution is listed below:

```
b = 0.01;               % size (in meters) of outer radius
er = 1:.1:10;           % range of relative permittivities
e0 = 8.85e-12;          % free space permittivity
u0 = 4*pi*10^-7;        % free space permeability
Z0 = 50;                % target impedance of the transmission line)
alpha = 5000;           % cost of conductor per cubic meter
beta = 150 + 25*er;     % cost of dielectric per cubic meter

a = b*exp(-2*pi*Z0*(e0/u0)^.5*(er.^.5));
Cost = pi*(alpha * a.^2 + (b^2-a.^2).*beta);
figure(1); plot( er, a );
xlabel('\epsilon_r'); ylabel('r (meters)');
title('Conductor Radius');

figure(2); plot( er, Cost );
xlabel('\epsilon_r'); ylabel('Cost ($/m)');
title('Coaxial Material Cost');
```