TDT3: Transmission Line Equations

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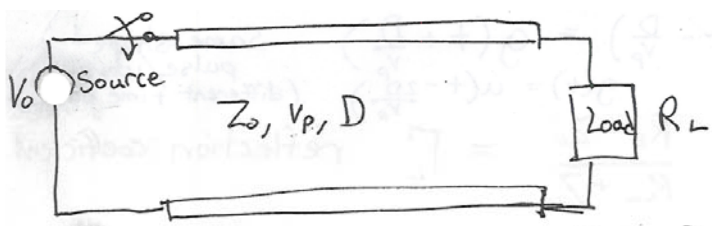
Start with a Discharged Transmission Line



$$v(z,t) = V^+ f\left(t - \frac{z}{v_p}\right) + V^- g\left(t + \frac{z}{v_p}\right)$$

$$i(z,t) = \frac{V^+}{Z_o} f\left(t - \frac{z}{v_p}\right) - \frac{V^-}{Z_o} g\left(t + \frac{z}{v_p}\right)$$

Uncharged Line $t < 0$

$v(z,t) = 0$ $\qquad$ $i(z,t) = 0$

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Switch in DC Source at t=0

Switched: $0 \le t \le D \rightarrow$ only forward wave

$$v(z,t) = V_0\, u\left(t - \frac{z}{v_p}\right) \qquad i(z,t) = \frac{V_0}{Z_0}\, u\left(t - \frac{z}{v_p}\right)$$

at the end

$$v(D,t) = V_0\, u\left(t - \frac{D}{v_p}\right) + \underbrace{V^- g\left(t + \frac{D}{v_p}\right)}_{0 \text{ until } t>D} = V_L$$

$$i(D,t) = \frac{V_0}{Z_0}\, u\left(t - \frac{D}{v_p}\right) - \frac{V^-}{Z_0}\, g\left(t + \frac{D}{v_p}\right) = I_L$$

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Reflection at the End of the Line

- Evaluate end-of-line boundary condition
- Note that left-hand expression is equal to a constant
- Conclusions
  - Backwards-traveling waveform also step function
  - Reflection is a function of load resistance

$$\frac{V^+ u\left(t - \frac{D}{v_p}\right) + V^- g\left(t + \frac{D}{v_p}\right)}{V^+ u\left(t - \frac{D}{v_p}\right) - V^- g\left(t + \frac{D}{v_p}\right)} = \frac{R_L}{Z_0} \;*$$

$$V^+ u\left(t - \frac{D}{v_p}\right)\left[1 - \frac{R_L}{Z_0}\right] = -\left[1 + \frac{R_L}{Z_0}\right] V^- g\left(t + \frac{D}{v_p}\right)$$

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Voltage as a Function of Space and Time

So we say $\qquad 2/5$

$$u\left(t-\frac{D}{v_p}\right) = g\left(t+\frac{D}{v_p}\right)$$

$$g(t) = u\left(t-\frac{2D}{v_p}\right)$$

Same-shaped pulse/wavefm (different time of [illegible])

$$\frac{V^-}{V^+} = \frac{R_L - Z_o}{R_L + Z_o} = \Gamma_L \quad \text{reflection coefficient}$$

so $T \leq t \leq 2T$

$$v(t,z) = V_o\, u\left(t-\frac{z}{v_p}\right) + V_o \Gamma_L\, u\left(t+\frac{z}{v_p}-\frac{2D}{v_p}\right)$$

$$i(t,z) = \frac{V_o}{Z_o} u\left(t-\frac{z}{v_p}\right) \quad \frac{V_o \Gamma_L}{Z_o} u\left(t+\frac{z}{v_p}-\frac{2D}{v_p}\right)$$

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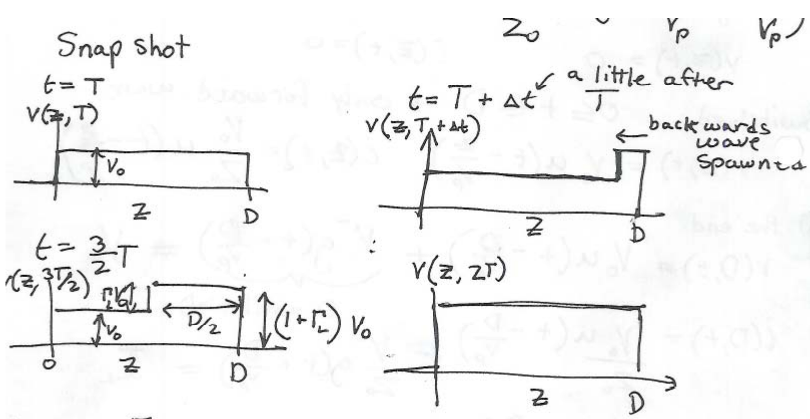
Voltage as a Function of Position on Line



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Notes about Reflection Coefficients

Notes: Γ_L can be positive or negative

$Z_0 = R_L$ is matched $\Gamma_L = 0$ $\qquad |\Gamma_L| \le 1$ for passive loads

$R_L < Z_0 \quad \Gamma_L < 0$

$R_L > Z_0 \quad \Gamma_L > 0$

$R_L = 0 \quad \Gamma_L = -1 \quad$ short

$R_L = \infty \quad \Gamma_L = +1 \quad$ open

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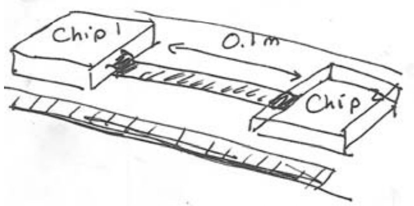
Sample Calculation – Problem Statement

Example $\qquad$ 5/5

Two digital chips are connected via a 50-Ω microstrip line with $v_p = 1 \times 10^8$ m/s and $D = 0.1$ m.



The digital chips use Emitter-Coupled Logic (ECL) which is high frequency, high power logic gate connection at 5 volts.

The input impedance of chip 2 is 100Ω while the source impedance of Chip 1 is 10Ω. If the line is uncharged (carrying a 0V charge) for a long period of time) and chip 1 suddenly outputs +5V logic state, how do the voltages propagate through the system?

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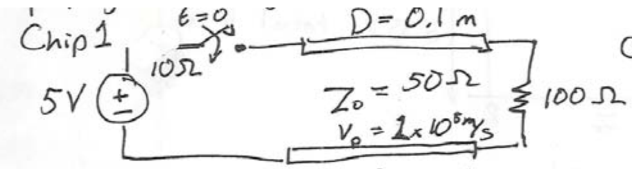
Sample Calculation – Setup



$$T = \frac{D}{v_0} = 1\times 10^{-9}\,\mathrm{s} = 1\,\mathrm{ns}$$

$$\Gamma_L = \frac{100-50}{100+50} = \frac{1}{3}$$

$$\Gamma_G = \frac{10-50}{50+10} = \frac{-2}{3}$$

$\boxed{t \le 0}$ Uncharged Line

$V(z,t) = 0 \qquad i(z,t) = 0$

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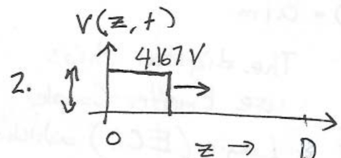
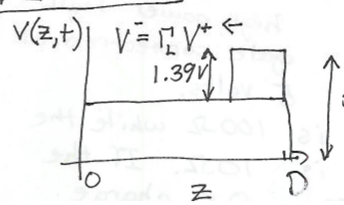
Sample Calculation – Solution to 2 Transit Times

$\boxed{0 \le t \le T}$ Initial Charge



$\boxed{T \le t \le 2T}$



5.55 V total at the load

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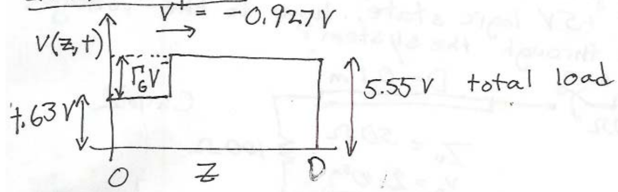
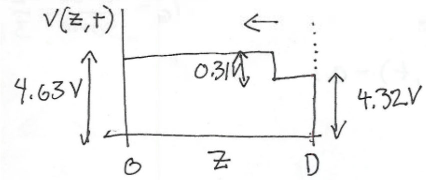
Sample Calculation – Solution to 4 Transit Times

$2T \leq t \leq 3T$



$3T \leq t \leq 4T$



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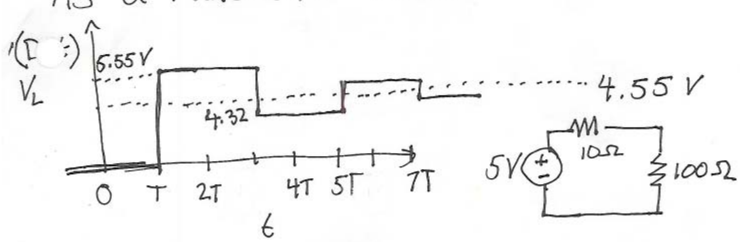
Sample Calculation – Temporal Output

As a function of time:



Tell-Tale "Ringing" at low Frequencies

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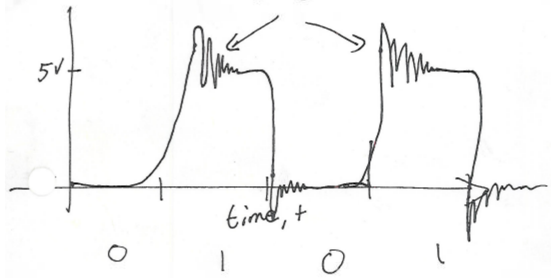
Ringing and other Reflection Effects

Tell-Tale "Ringing" at low Frequencies



Intersymbol Interference
Load Transformation

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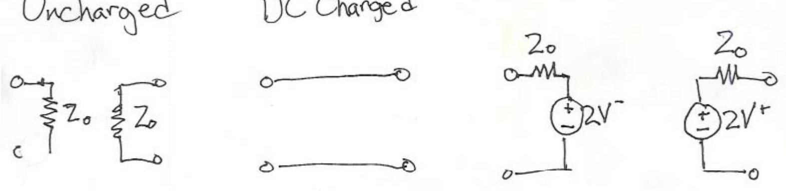
Equivalent Circuits for Transmission Lines

Eq. Circuit

Uncharged

DC Charged



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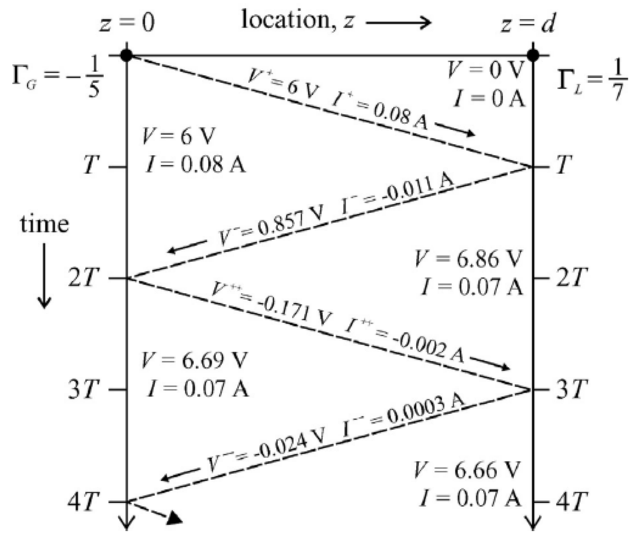
Anatomy of a Bounce Diagram



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