Curriculum Topic: 
Time-Domain Transmission Lines

TDT5: Cascades and Fan-Outs

Module Outline:

<table>
<thead>
<tr>
<th>Prerequisite Skills</th>
<th>Competencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplemental Reading and Resources</td>
<td>Assessments</td>
</tr>
<tr>
<td>Laboratory Activities</td>
<td>Power Point Slides and Notes</td>
</tr>
</tbody>
</table>

Prerequisite Skills

Prerequisites / Requirements:
TDT4 Termination Schemes

Competencies

Competency TDT.5: Calculate reflections and track signals when more than one transmission line is in a circuit network.

Competency Builders:
TDT.5.1 Calculate the reflection coefficient at the junction of cascaded transmission lines.
TDT.5.2 Calculate the reflection coefficient at the junction of a transmission line fan-out.
TDT.5.3 Track signal flow in a circuit with multiple transmission lines.

Supplemental Reading and Resources

Supplemental Reading Materials:
Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

Question: TDT5.1  Competency: TDT.5.1
Three transmission lines are cascaded together, the first one with impedance $Z_0$, the second with impedance $Z_1$, and the third with impedance $Z_2$. What is the impedance of the middle section $Z_1$ in terms of the other two impedances that maximizes the amplitude of the initial voltage of a DC pulse traveling on the last transmission line segment?

Answer:

When the pulse hits the junction of lines 1 and 2, it transmits with coefficient $\tau_{01}$ into line 2. When the pulse hits the junction of lines 2 and 3, it transmits with coefficient $\tau_{12}$ into line 3. Mathematically, these transmission coefficients are

\[
\tau_{01} = \frac{2Z_1}{Z_0 + Z_1}, \quad \tau_{12} = \frac{2Z_2}{Z_1 + Z_2}
\]

To figure out the maximum voltage, we maximize the product $\tau_{01}\tau_{12}$ with respect to $Z_1$. The value of impedance that maximizes the voltage amplitude is

\[
Z_1 = \sqrt{Z_0Z_2}
\]

Question: TDT5.2  Competency: TDT.5.2
If a source transmission line with impedance $100 \, \Omega$ is connected in parallel to 4 electrically identical lines, what should be their impedances to minimize reflections at the junction? If the 4 lines are connected in series fan-out, what should their impedances be?

Answer:

$400 \, \Omega, 25 \, \Omega$

Question: TDT5.3  Competency: TDT.5.3
Two mismatched transmission lines are cascaded together with a switched DC-source and resistive load that are also mismatched. If the transit time of the first line is $1.2$ ns and the transit time of the second line is $.5$ ns, write down the first 12 times that you will see voltage changes at the output if the DC switch is turned on at $t=0$.

Answer:
Key: 1-2-2 is shorthand for a signal with leading edge that travels down transmission line 1 ($T_1$ transit time), then transmission line 2 (back and forth for $+2T_2$ transit time), and finally down transmission line 1 again to the load ($+T_2$ transit time).

Total Transit Time =
1.7 ns (1-2),
2.7 ns (1-2-2),
3.7 ns (1-2-2-2),
4.1 ns (1-1-2),
4.7 ns (1-2-2-2-2),
5.1 ns (1-2-1-2 and 1-1-2-2),
5.7 ns (1-2-2-2-2-2),
6.1 ns (1-1-2-2-2 and 1-2-1-2),
6.7 ns (1-2-2-2-2-2-2),
7.1 ns (1-1-2-2-2-2 and 1-2-1-2-2-2),
7.5 ns (1-2-1-1-2),
7.7 ns (1-2-2-2-2-2-2-2)