Curriculum Topic :
Time-Domain Transmission Lines

TDT5 : Cascades and Fan-Outs

Module Outline:	
Prerequisite Skills	Competencies
Supplemental Reading and Resources	Assessments
Laboratory Activities	Power Point Slides and Notes

Prerequisite Skills

Prerequisites / Requirements:
TDT4 Termination Schemes

Competencies

Competency TDT.5: Calculate reflections and track signals when more than one transmission line is in a circuit network.

Competency Builders:
TDT.5.1 Calculate the reflection coefficient at the junction of cascaded transmission lines.
TDT.5.2 Calculate the reflection coefficient at the junction of a transmission line fan-out.
TDT.5.3 Track signal flow in a circuit with multiple transmission lines.

Supplemental Reading and Resources

Supplemental Reading Materials:

A.F. Peterson and G.D. Durgin. *Transient Signals on Transmission Lines: An Introduction to the Non-Ideal Effects and Signal Integrity Issues in Electrical Systems*. Morgan & Claypool Publishers, 2009. Chapter 5.

Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

Question: TDT5.1 *Competency:* TDT.5.1

Three transmission lines are cascaded together, the first one with impedance Z_0, the second with impedance Z_1, and the third with impedance Z_2. What is the impedance of the middle section Z_1 in terms of the other two impedances that maximizes the amplitude of the initial voltage of a DC pulse traveling on the last transmission line segment?

Answer:

When the pulse hits the junction of lines 1 and 2, it transmits with coefficient τ_{01} into line 2. When the pulse hits the junction of lines 2 and 3, it transmits with coefficient τ_{12} into line 3. Mathematically, these transmission coefficients are

$$\tau_{01} = \frac{2Z_1}{Z_0 + Z_1} \qquad \tau_{12} = \frac{2Z_2}{Z_1 + Z_2}$$

To figure out the maximum voltage, we maximize the product $\tau_{01}\tau_{12}$ with respect to Z_1. The value of impedance that maximizes the voltage amplitude is

$$Z_1 = \sqrt{Z_0 Z_2}$$

Question: TDT5.2 *Competency:* TDT.5.2

If a source transmission line with impedance 100 Ω is connected in parallel to 4 electrically identical lines, what should be their impedances to minimize reflections at the junction? If the 4 lines are connected in series fan-out, what should their impedances be?

Answer:

400 Ω, 25 Ω

Question: TDT5.3 *Competency:* TDT.5.3

Two mismatched transmission lines are cascaded together with a switched DC-source and resistive load that are also mismatched. If the transit time of the first line is 1.2 ns and the transit time of the second line is .5 ns, write down the first 12 times that you will see voltage changes at the output if the DC switch is turned on at t=0.

Answer:

Key: 1-2-2 is shorthand for a signal with leading edge that travels down transmission line 1 (T_1 transit time), then transmission line 2 (back and forth for $+2T_2$ transit time), and finally down transmission line 1 again to the load ($+T_2$ transit time).
Total Transit Time =
1.7 ns (1-2),
2.7 ns (1-2-2),
3.7 ns (1-2-2-2),
4.1 ns (1-1-2),
4.7 ns (1-2-2-2-2),
5.1 ns (1-2-1-2 and 1-1-2-2),
5.7 ns (1-2-2-2-2-2),
6.1 ns (1-1-2-2-2 and 1-2-1-2-2),
6.7 ns (1-2-2-2-2-2-2),
7.1 ns (1-1-2-2-2-2 and 1-2-1-2-2-2),
7.5 ns (1-2-1-1-2),
7.7 ns (1-2-2-2-2-2-2-2)