Curriculum Topic : Time-Domain Transmission Lines

TDT6 : Initially Charged Transmission Lines

Module Outline:	
Prerequisite Skills	Competencies
Supplemental Reading and Resources	Assessments
Laboratory Activities	Power Point Slides and Notes

Prerequisite Skills

Prerequisites / Requirements:
TDT5 Termination Schemes

Competencies

Competency TDT.6: **Analyze the transient behavior of a charged transmission line**

Competency Builders:
TDT.6.1 Calculate the voltages and currents on a DC-charged line with a switched source.
TDT.6.2 Calculate the initial transient and steady states of charged transmission lines in more complicated resistive networks.

Supplemental Reading and Resources

Supplemental Reading Materials:

A.F. Peterson and G.D. Durgin. *Transient Signals on Transmission Lines: An Introduction to the Non-Ideal Effects and Signal Integrity Issues in Electrical Systems*. Morgan & Claypool Publishers, 2009. Chapter 6.

Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

Question: TDT6.1 *Competency:* TDT.6.1

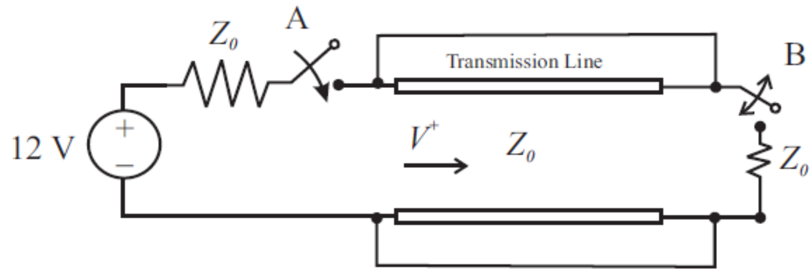
An open-circuited 200-Ω transmission line is energized to 6 V DC across the load terminal. What is the total load output voltage and backward-propagating voltage (as measured at the end of the line) immediately after an 80-Ω resistor is switched into the load terminals?

Answer:

Under steady-state DC excitation, there is a +3V waveform traveling both in the forward and backwards directions. When the 80-Ω resistor is switched into the load, the reflection coefficient changes from +1 to -3/7. This means that the total output voltage will become +1.71 V and the backward propagating voltage will become -1.29 V immediately after the switch is thrown.

Question: TDT6.2 *Competency:* TDT.6.2

T-line Sequence Problem: Below is a transmission line circuit. Fill out the state table for V^+ as the circuit is sequentially switched under the following conditions. Note: V^+ is measured at the *start* of the transmission line. Assume ideal circuit components. (**22 points**)



	V^+	Condition
State 0		Both switches are open and the t-line is discharged
State 1		Immediately after switch A is closed
State 2		Switch A has been closed for a while
State 3		Immediately after switch A is opened
State 4		Switch A has been open for a while
State 5		Immediately after switch B is closed
State 6		Switch B has been closed for a while
State 7		Immediately after switch A is closed
State 8		Switch A has been closed for a while
State 9		Immediately after switch B is opened
State 10		Switch B has been opened for a while

Answer:

State 0 0 V; system is uncharged.

State 1 4 V; a forward and a backward-traveling wave of 4 V are launched simultaneously from both ends of the line.

State 2 6 V; steady-state circuit shows effectively an open circuit voltage of 12 V and both ends and no current flowing into or out of the line. $V^+ = (V_L + I_L Z_0)/2 = 6$ V, which is also V^-.

State 3 6 V; the system is in equilibrium, current still does not (cannot) flow between transmission line ports.

State 4 6 V; this state persists as there is no resistor to discharge across, no connection from top plate to bottom. Looks like a parallel plate capacitor.

State 5 2 V; there is now a load voltage of 8 V and a current $I_L = 4/Z_0$ flowing across the terminals of both sides of the transmission line. On the load side, $V^+ = (V_L + I_L Z_0)/2 = 6$ V and $V^- = (V_L - I_L Z_0)/2 = 2$ V. This behavior is mirrored on the source side, except V^+ and V^- are flipped.

State 6 0 V; completely discharged.

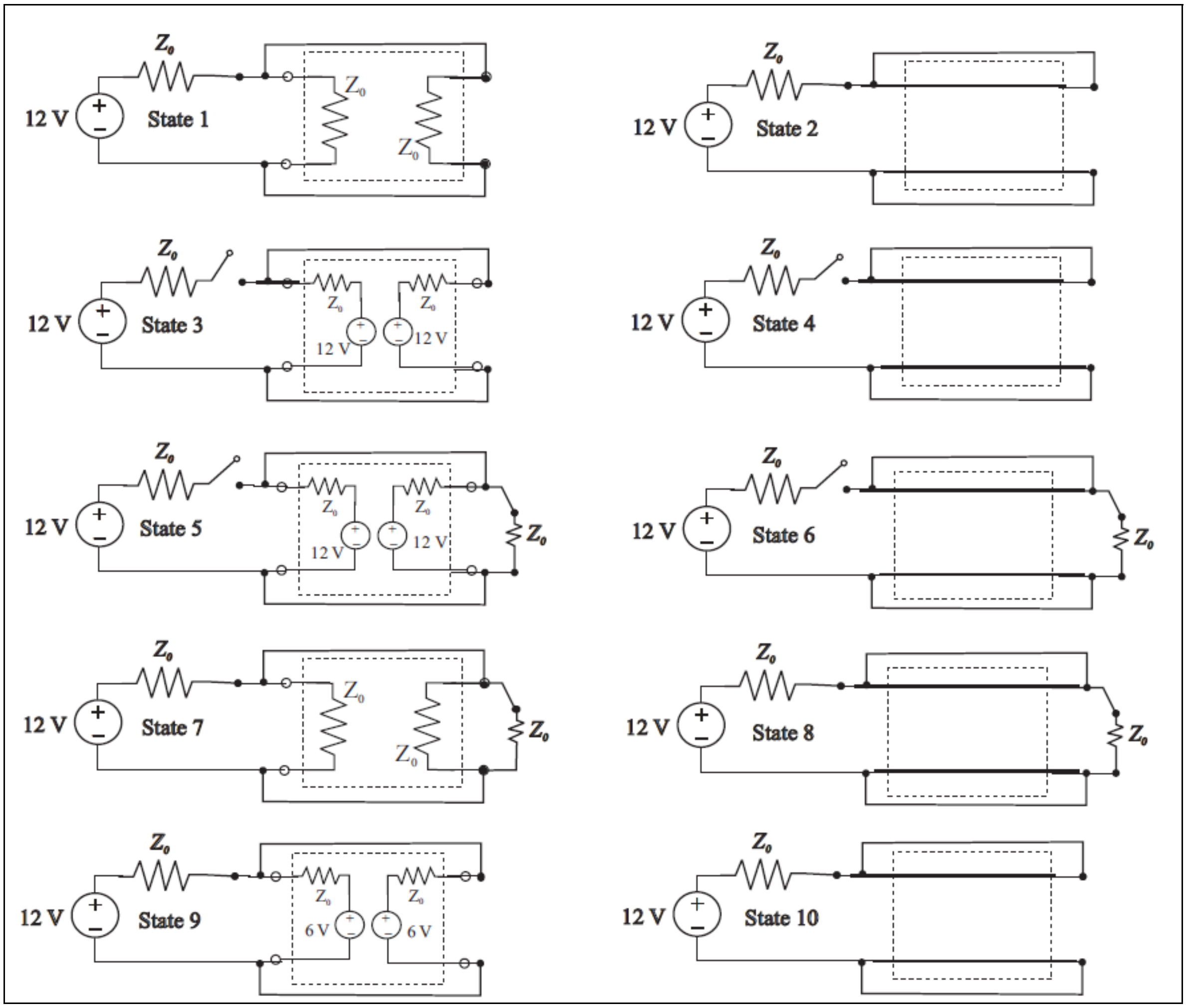
State 7 3 V; we can use the discharged model for the transmission line. A forward and a backward-traveling wave of 3 V are launched simultaneously from both ends of the line.

State 8 3 V; the forward and backward propagating waves are identical at steady-state and must add up to 6 V.

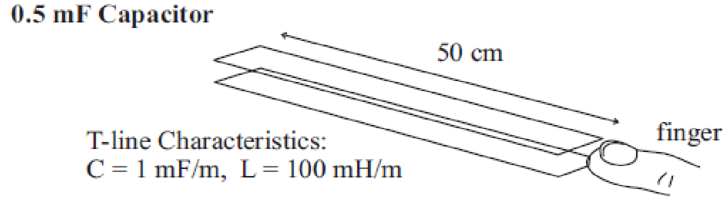
State 9 5 V; there is a total of 8 V across both ends of the line and a current of $-2/Z_0$ flowing across both the terminals of the line. On the load side, $V^+ = (V_L + I_L Z_0)/2 = 3$ V and $V^- = (V_L - I_L Z_0)/2 = 5$ V. This behavior is mirrored on the source side, except V^+ and V^- are flipped.

State 10 4 V; identical to state 2.

The equivalent circuits used for this problem are shown below:

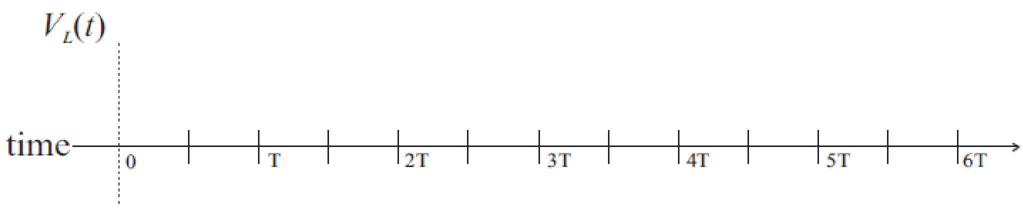

Z₀
12 V
State 1
Z₀
Z₀
State 2
State 3
12 V
State 4
State 5
State 6
State 7
State 8
State 9
6 V
State 10

Discharge of a Long Capacitor: As a practical joke, you charge up a long, skinny parallel-plate capacitor to 200 V and leave it in your friend's sock drawer. While charged, your friend begins rummaging for socks and touches his 190Ω finger to the end of the capacitor, which can be modeled as a transmission line with intrinsic impedance of $Z_0 = 10\Omega$:

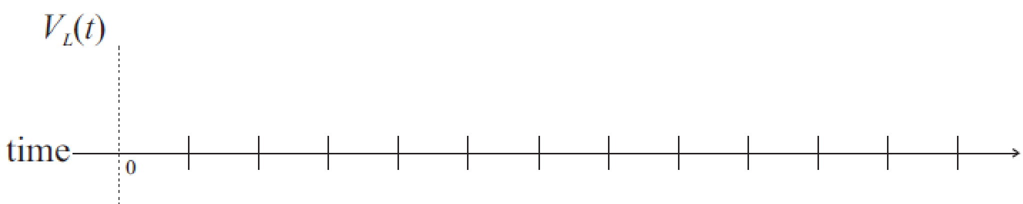


Answer the following questions based on this scenario. (**25 points**)

(a) Sketch the total voltage across the finger as a function of time in the graph below. You do not need to label amplitudes. (**10 points**)



(b) If the capacitor was modeled as a single lumped-parameter circuit (a resistive finger in parallel with a charged capacitor) sketch what the transient would look like. (**10 points**)

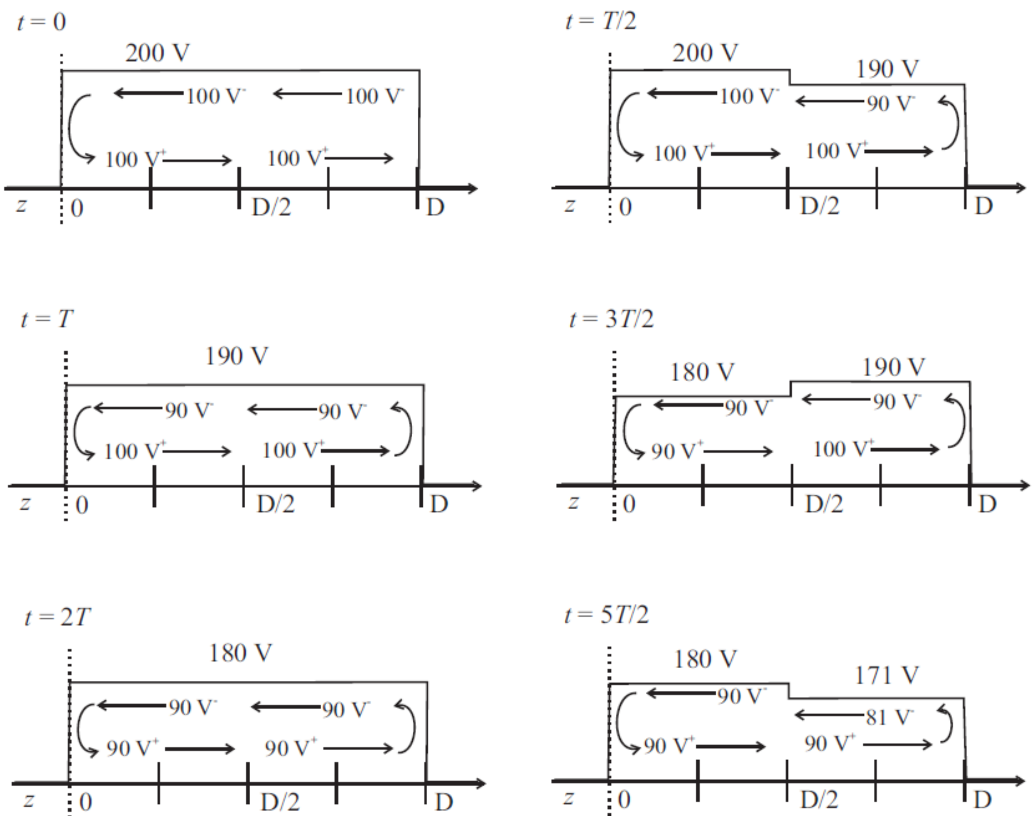


(c) Under what condition(s) would the transient graph in part (a) resemble the transient graph in part (b)? (**5 points**)

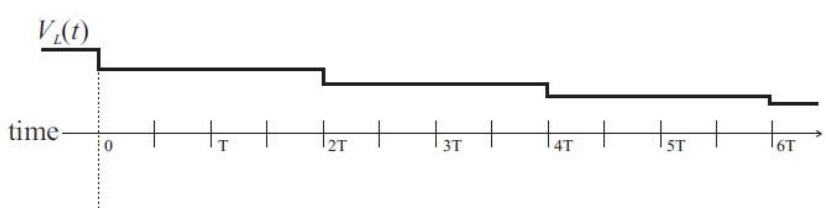
(d) (**Bonus +5 Points**): Show mathematically how the voltage transient in (b) becomes equivalent to the voltage transient in (a) under certain conditions and identify which conditions lead to this (in terms of C, L, Z_0, D, and/or T). No partial credit. Do not attempt this unless the rest of the test has been completed to your satisfaction.

Answer:

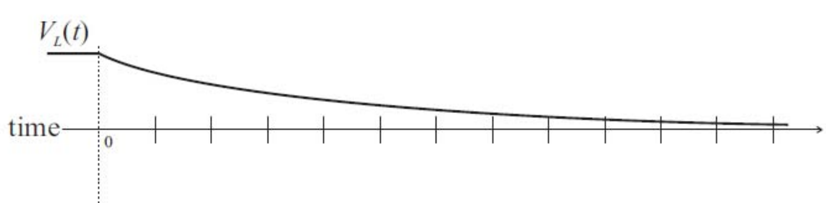
Discharge of a Long Capacitor: This problem is identical to the homework problem, only now the long, skinny capacitor is mismatched to the finger, with a reflection coefficient of $\Gamma_L = 0.9$. As such, it will take a lot longer than $2T$ to discharge because a diminishing voltage waveform will continue to circulate around the line, gradually stepping down the total load voltage:



(a) Each step-down in voltage occurs during even multiples of a transit time:



(b) A charged, lump-sum capacitor has simply an exponential discharge across the resistive finger:



(c) The graphs in part (a) will resemble the graph in part (b) if either of the following conditions are met: the capacitor is relatively short (small T) or the line is much more capacitive than inductive (small Z_0). Either of these conditions will cause the steps in part (a) to appear small and/or drawn out in time, so that it will simply look like a granular version of exponential decay.

(d) (**Bonus +5 Points**): This was a tough one, but a great conceptual exercise to recognize that the RC-characteristic you studied in circuits class is, under certain conditions, the same solution that we study in this class! The lumped parameter RC circuit discharges with the following characteristic for $t \geq 0$:

$$V(t) = V_0 \exp\left(-\frac{t}{R_L C'}\right) = V_0 \exp\left(-\frac{t}{R_L C D}\right)$$

where C' is the total capacitance, which is related to the per-unit-length capacitance, C ($C' = CD$). In the transmission line model, there is a geometrically step-decaying solution which can be expressed as

$$V(t) = V_0/2 \underbrace{\left(1 + \frac{R_L - Z_0}{R_L + Z_0}\right)}_{\tau_L} \underbrace{\left(\frac{R_L - Z_0}{R_L + Z_0}\right)}_{\Gamma_L}^{\lfloor \frac{t}{2T} \rfloor}$$

where $\lfloor \cdot \rfloor$ denotes the floor (round down) operation, ensuring that another reflection coefficient is added to the expression every $2T$ to achieve that stair-stepped decay. If the skinny capacitor has a small transmit time, then we can approximate this voltage as

$$V(t) \approx V_0/2 \left(1 + \frac{R_L - Z_0}{R_L + Z_0}\right) \left(\frac{R_L - Z_0}{R_L + Z_0}\right)^{\frac{t}{2DZ_0C}}$$

which takes use of the relationship $T = D\sqrt{LC} = DZ_0C$. Now, if the line is much more capacitive than inductive, $Z_0 = \sqrt{\frac{L}{C}}$ becomes very small. Let's take that limit:

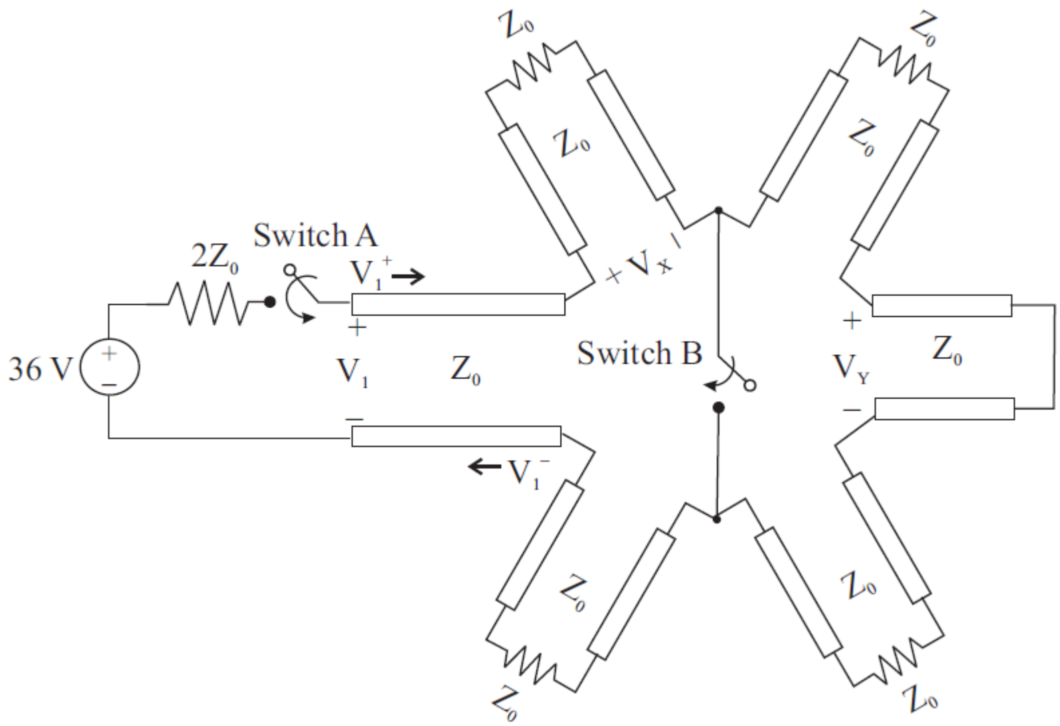
$$V(t) \approx V_0/2 \underbrace{\left(1 + \frac{R_L - Z_0}{R_L + Z_0}\right)}_{\lim_{Z_0 \to 0} = 2} \left[\underbrace{\left(\frac{R_L - Z_0}{R_L + Z_0}\right)^{\frac{1}{Z_0}}}_{\lim_{Z_0 \to 0} = \exp\left(-\frac{2}{R_L}\right)}\right]^{\frac{t}{2DC}} \approx V_0 \exp\left(-\frac{t}{R_L DC}\right)$$

It's our original RC-constant solution for a lumped-sum capacitor! Under conditions of small Z_0 and/or small T, these graphs would look identical on an oscilloscope!

Question: TDT6.4 *Competency:* TDT.6.2

(3) **The Death Star:** The circuit below represents a high-speed digital interconnect that is switched according to the following states:

- State 0: Both switches are open and both lines are uncharged.
- State 1: Immediately after switch A is closed onto the DC source.
- State 2: Switch A has been closed for a while.
- State 3: Immediately after switch B is closed.
- State 4: Switch B has been closed for a while.
- State 5: Immediately after Switch B re-opens.



Fill out the following table according to these switching states. Unsimplified fractional answers are OK. Assume all backwards propagating waves are measured from the right-most side of the transmission line. Assume all forward propagating waves are measured from the left-most side of the transmission line. (**44 points**):

	V_1	V_X	V_Y	V_1^+	V_1^-
State 0	0	0	0	0	0
State 1		0	0		0
State 2					
State 3					
State 4					
State 5					

Answer:

	V_1	V_X	V_Y	V_1^+	V_1^-
State 0	0	0	0	0	0
State 1	12	0	0	12	0
State 2	24	6	0	15	9
State 3	24	10	-8	15	5
State 4	18	9	0	27/2	9/2
State 5	18	9/2	9/2	27/2	9