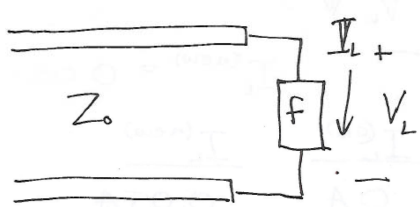
TDT9: Nonlinear Loads on Transmission Lines

By Prof. Gregory D. Durgin

Reflection Calculation of an Arbitrary Load



$$V^+ = \frac{V_L + I_L Z_0}{2} \qquad V^- = \frac{V_L - I_L Z_0}{2}$$

$$I_L = \frac{2}{Z_0} V^+ - \frac{V_L}{Z_0}$$

Iteration for a Non-linear Load

for resistive case:

$$I_L = V_L / R_L \Rightarrow V_L = I_L R_L$$

$$I_L = \frac{2}{Z_0} V^+ - I_L \frac{R_L}{Z_0}$$

$$I_L^{(new)} = \frac{2}{Z_0} V^+ - I_L^{(old)} \frac{R_L}{Z_0}$$

① make a guess for $I_L^{(old)}$
② Calculate $I_L^{(new)}$ from $I_L^{(old)}$
③ if $I_L^{(new)} \neq I_L^{(old)}$, set $I_L^{(old)} = I_L^{(new)}$ and repeat ②

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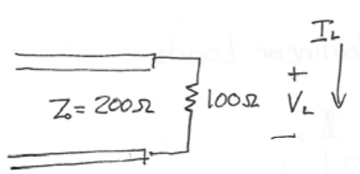
Iteration with a Resistive Load



$V^+ \rightarrow 5\,V$

$$I_L^{(new)} = 0.05 - \frac{1}{2} I_L^{(old)}$$

Guess #	$I_L^{(old)}$	$I_L^{(new)}$
1	0 A	0.05 A
2	0.05A	0.025A
3	0.025A	0.0375A
4	0.0375A	0.03125A
5	0.03125A	0.034 A
6	0.034 A	0.033 A
7	0.033 A	0.034A
8	0.034 A	0.033 A
9	0.033 A	0.033A

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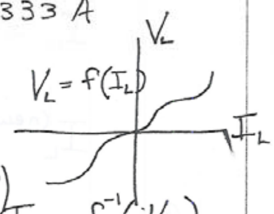
Strategy for Nonlinear Loads

I_L should converge to 0.03333 A

Other form:

$$V_L^{(new)} = 2V^+ - I_L Z_0$$



$$\rightarrow V_L^{(new)} = 2V^+ - Z_0 f^{-1}(V_L^{(old)}) \qquad I_L = f^{-1}(V_L)$$

$$\rightarrow I_L^{(new)} = \frac{2}{Z_0}V^+ - \frac{1}{Z_0} f(I_L^{(old)})$$

Pick 1 or the other based on stability

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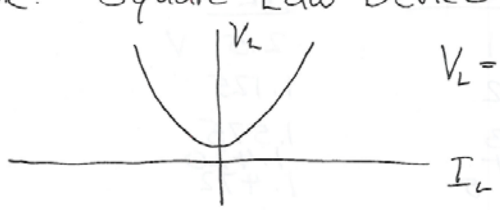
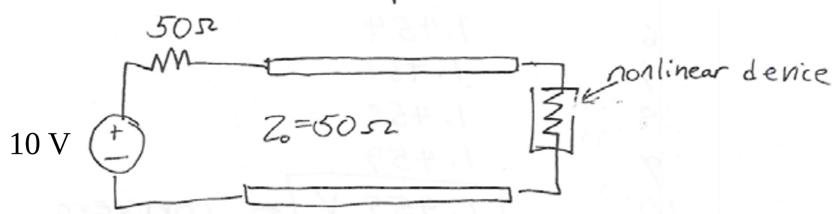
Example with Nonlinear Load

Example: Square Law Device





$V_L = 50 I_L^2$

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Setup Voltage Iteration

$$V_L = 50 I_L^2$$

$$I_L = (2V^+)/Z_0 - V_L/Z_0$$

① $$I_L^2 = \frac{[2V^+ - V_L]^2}{Z_0^2}$$

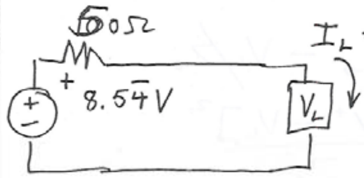
$$V_L = \frac{[2V^+ - V_L]^2}{50}$$

$$V_L^{new} = \frac{[2V^+ - V_L^{old}]^2}{50}$$

Sample Iteration

Iter #	V_L
1	2.5 V
2	1.125
3	1.575
4	1.420
5	1.472
6	1.454
7	1.461
8	1.458
9	1.459
10	1.459 V ← convergence

Steady-State Result



$I_L = 0.1708$ A

$V_L = 1.459$

Reflections?

$$V^- = \frac{V_L - Z_0 I_L}{2}$$

$$= \frac{1.459 - 50(0.1708)}{2}$$

$$= -3.54\ V$$

$$V^+ = 5\ V$$

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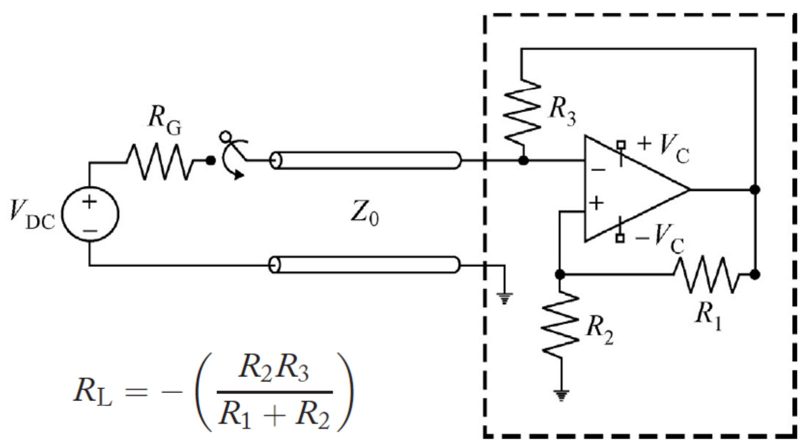
Brain Teaser



$$R_L = -\left(\frac{R_2 R_3}{R_1 + R_2}\right)$$

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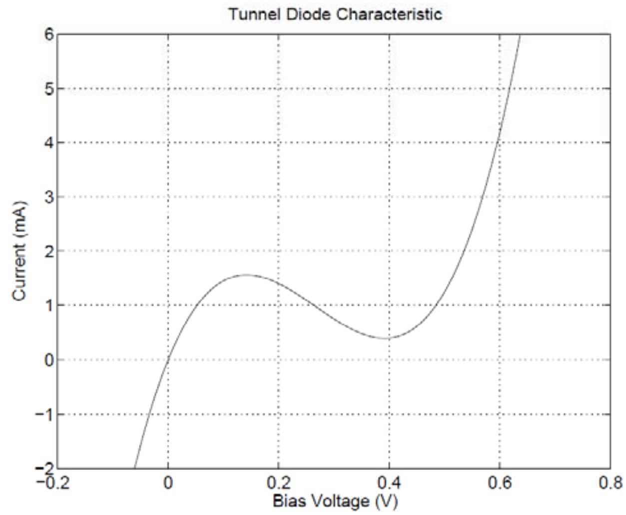
Tunnel Diode



Tunnel Diode Characteristic: $I = 150V^3 - 120V^2 + 25V$ mA

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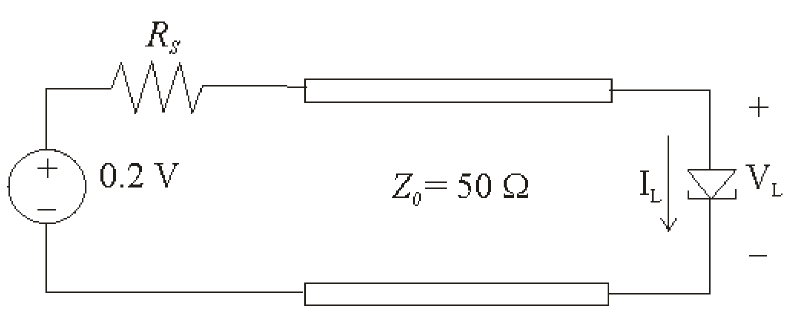
Transmission Line Terminated with Tunnel Diode



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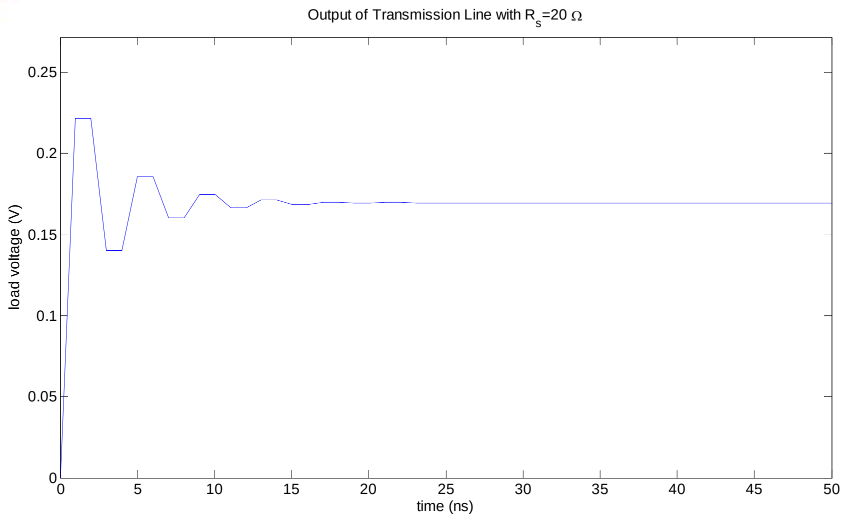
Driving at R_s = 20 Ω



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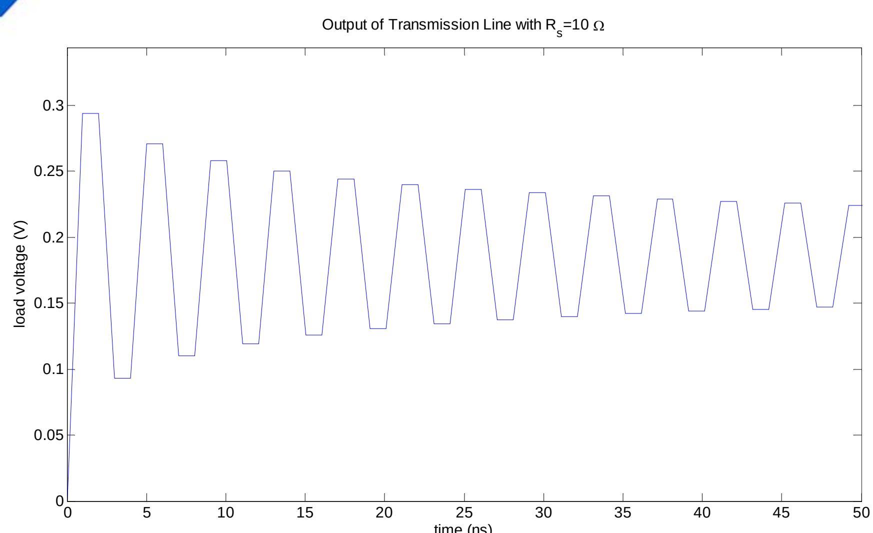
Driving at R_s = 10 Ω



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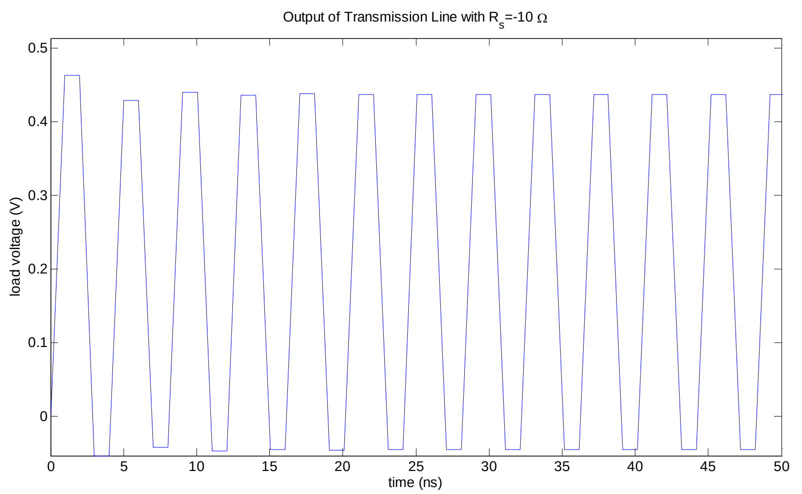
Overdriving at R_s = -10 Ω



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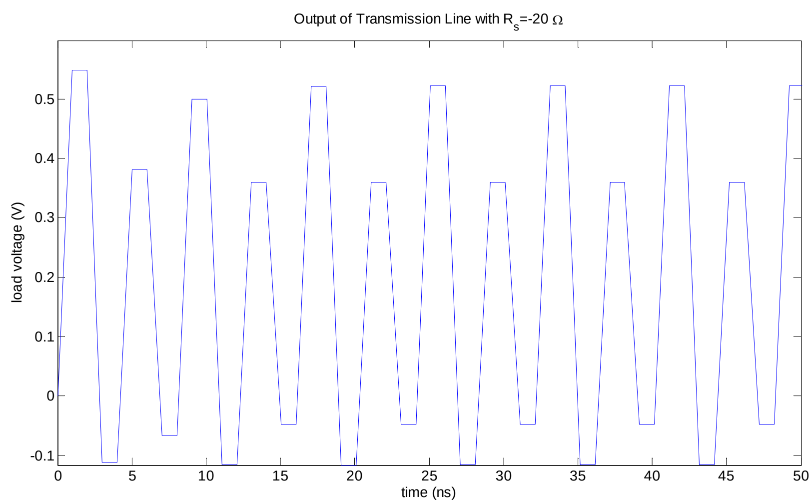
Overdriving at R_s = -20 Ω



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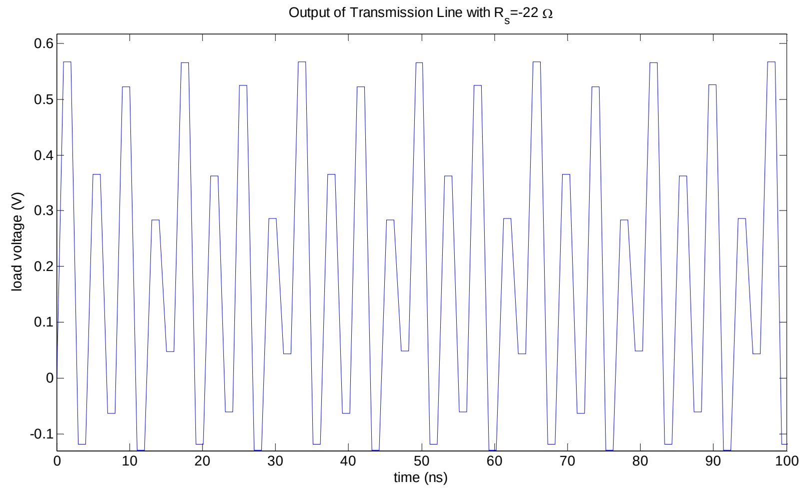
Overdriving at R_s = -22 Ω



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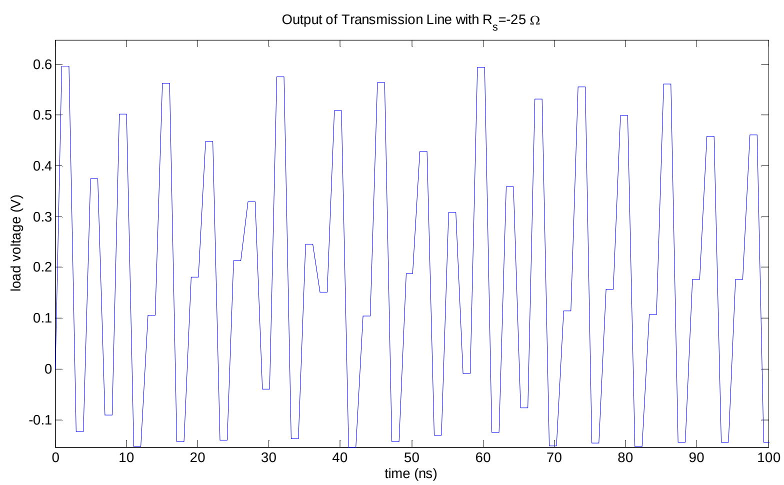
Chaotic Behavior at R_s = -25 Ω



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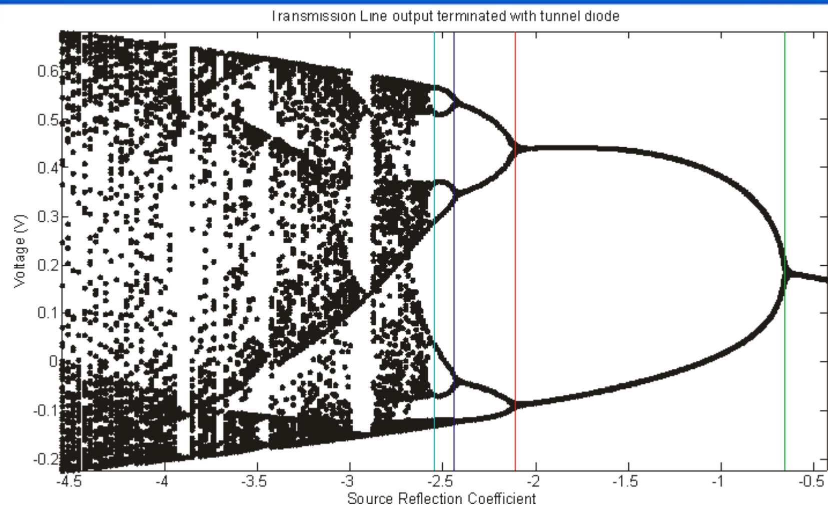
Output Voltages vs. Source Reflection Γ_s



δ=4.669201609102990671853203820...

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Attributes of Chaotic Circuits and Systems

- Ingredients for chaos in circuits
  - Nonlinear element in the circuit
  - Active component (source/negative resistor)
  - Third order system (at least 3 caps/inductors)
  - Example: Chua's circuit
- Attributes
  - Stable point bifurcation, followed by chaotic region
  - Islands of stability, odd-valued bifurcations
  - Bifurcation period tends to Feigenbaum's constant (4.6692…) in the limit approaching chaos

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For Further Reading

- L.O. Chua and G.-N. Lin. “Canonical Realization of Chua’s Circuit Family,” *IEEE Journal of Circuits and Systems*. vol 37, no 7, July 1990, pp 885-902.
- L. Corti, L. De Menna, G. Miano, L. Verolino. “Chaotic Dynamics in an Infinite-dimensional Electromagnetic System,” IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications, vol 41, no 11, Nov 1994, pp 730-736.