Curriculum Topic: Time-Harmonic Transmission Lines

THT2: Sinusoids on Transmission Lines

Module Outline:

<table>
<thead>
<tr>
<th>Prerequisite Skills</th>
<th>Competencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplemental Reading and Resources</td>
<td>Assessments</td>
</tr>
<tr>
<td>Power Point Slides and Notes</td>
<td></td>
</tr>
</tbody>
</table>

Prerequisite Skills

Prerequisites / Requirements:

THT1 Phasor Review

Competencies

Competency THT.2: Describe the mathematical form of sinusoidal waves on a transmission line

Competency Builders:

THT.2.1 Write the voltage and current solutions for a time-harmonic transmission line in the phasor domain

THT.2.2 Quantify the relationship between wavenumber, wavelength, and frequency

Supplemental Reading and Resources

Supplemental Reading Materials:

Prof. Peterson’s online lecture notes 13
Assessments

The following questions and exercises may serve as either pre-assessment or post-assessment tests to evaluate student knowledge.

**Question:** THT.2.1  
**Competency:** THT.2.1

True or False: Under steady-state time-harmonic excitation, a useful equivalent circuit for a transmission line is two short circuit pathways.

**Answer:** false

**Question:** THT.2.2  
**Competency:** THT.2.2

A transmission line with velocity of propagation $1.7 \times 10^8$ m/s is excited at 2.45 GHz. What is the wavenumber of this line?

**Answer:** 90.6 rad/m

**Question:** THT.2.3  
**Competency:** THT.2.1

Transmission Line with Sinusoidal Excitation: (30 points) Below are phasor-domain voltage and current to a transmission line operating with steady-state sinusoids with frequency $f$:

$$v(z) = 100 \exp(-j\beta[z - D]) + 50 \exp(j\pi + j\beta[z - D])$$

$$i(z) = \exp(-j\beta[z - D]) - \frac{1}{2} \exp(j\pi + j\beta[z - D])$$

where $z = 0$ is the source side and $z = D$ is the load side. Perform the following analysis:

(a) In the equations above, circle the portion of the solution representing the backward-propagating current waveform. (5 points)

(b) In the equations above, box the forward-propagating amplitude of the voltage waveform. (5 points)

(c) Write a simplified expression for time-varying voltage evaluated at the front of the line ($z = 0$). Your answer should be in the time domain, $v(t)$. (5 points)

(d) What is the characteristic impedance, $Z_0$, of this line? (5 points)

(e) What is the VSWR of voltages on this line? (5 points)

(f) What is the load impedance, $Z_L$, for this line? (5 points)

**Answer:**
(a) In the equations above, circle the portion of the solution representing the backward-propagating current waveform. The term $\frac{1}{2} \exp(j\pi + j\beta[z - D])$ should be circled.

(b) In the equations above, box the forward-propagating amplitude of the voltage waveform. The amplitude 100 should be boxed.

(c) This follows from the phasor inverse transform definition (see formula sheet)

$$v(t, 0) = \text{Real} \{ \tilde{v}(0) \exp(j2\pi ft) \}$$
$$= 100 \cos(2\pi ft + \beta D) + 50 \cos(2\pi ft - \beta D + \pi)$$
$$= 100 \cos(2\pi ft + \beta D) - 50 \cos(2\pi ft - \beta D)$$

Anyone who made it to line two got full credit.

(d) $Z_0 = 100\Omega$

(e) There are two ways of doing this. First, we immediately see that $|\Gamma| = 0.5$ because the reflected voltage/current wave is half the magnitude of the forward voltage/current wave. This can be plugged directly into one of the VSWR formulas to get VSWR=3. The same result can be obtained from the following reasoning: VSWR is the ratio of max voltage to min voltage on the line; the max voltage occurs when the forward wave adds in phase with the backwards wave (100+50 V); the min voltage occurs when the forward wave adds out-of-phase (destructively) with the backwards wave (100-50 V); thus, the VSWR will be 150/50 or 3.

(f) For this problem, I gave full credit to anyone who put down line 1 of the following answer, but did not finish the calculation. Very few people got full credit for this problem.

$$Z_L = \frac{\tilde{v}(D)}{\tilde{v}(D)}$$
$$= \frac{100 + 50 \exp(j\pi)}{1 - \frac{1}{2} \exp(j\pi)}$$
$$= \frac{100 - 50}{1 + \frac{1}{2}}$$
$$= \frac{100}{3} \Omega$$

Many people tried to solve this from the VSWR by using the property $|\Gamma| = \frac{1}{2}$ to backsolve $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$. This actually doesn’t work because it only gives you the magnitude of the reflection coefficient. Many people worked the problem with $\Gamma = \frac{1}{2}$ and found $Z_L = 300\Omega$, when in fact the reflection coefficient for this particular example was $\Gamma = -\frac{1}{2}$. 