THT3: Open- and Short-Circuit Loads on Sinusoidal Tlines

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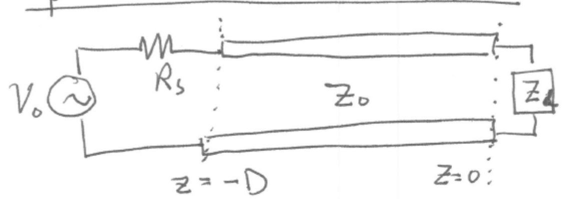
Sinusoids on Transmission Lines

Open + Short Circuit Loads



wavenumber

$\beta = \frac{2\pi}{\lambda}$

$$\tilde{V}(z) = \tilde{V}^+ \exp(-j\beta z) + \tilde{V}^- \exp(j\beta z)$$

$$\tilde{i}(z) = \frac{\tilde{V}^+}{Z_o} \exp(-j\beta z) - \frac{\tilde{V}^-}{Z_o} \exp(j\beta z)$$

Complex Reflection Coefficient

at end of line

$$\tilde{V}(0) = \tilde{V}^+ + \tilde{V}^-$$

$$\tilde{i}(0) = \frac{\tilde{V}^+}{Z_0} - \frac{\tilde{V}^-}{Z_0}$$

$$\frac{\tilde{V}(0)}{\tilde{i}(0)} = \tilde{Z}_L \Rightarrow \tilde{V}^- = \Gamma_L \tilde{V}^+$$

now complex reflection $\quad \Gamma_L = \dfrac{\tilde{Z}_L - Z_0}{\tilde{Z}_L + Z_0}$

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Open Circuit Analysis

Open Circuit

$$\Gamma_L = +1 \Rightarrow \tilde{V}^- = \tilde{V}^+$$

$$\tilde{V}(z) = 2\tilde{V}^+ \left(\frac{\exp(-j\beta z) + \exp(j\beta z)}{2} \right)$$

$$= 2\tilde{V}^+ \cos\beta z$$

$$\tilde{i}(z) = j2\frac{\tilde{V}^+}{Z_0} \Bigg(\underbrace{\frac{\exp(-j\beta z) - \exp(+j\beta z)}{j2}}_{-\sin\beta z} \Bigg)$$

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Thevenin Equivalent Impedance of an Open Line

Thevenin equivalent at front of line

$$\tilde{Z}_{th}(D) = \frac{\tilde{V}(z)\big|_{z=-D}}{\tilde{I}(z)\big|_{z=-D}} = -jZ_0 \cot \beta z$$

$\tilde{Z}_{th}(0) \Rightarrow$ open circuit, $+\infty$

$\tilde{Z}_{th}(D)$, $0 < D < \frac{\lambda}{4}$, $-jX_c$ capacitor

$\tilde{Z}_{th}(\lambda/4) = 0$, short circuit

$\tilde{Z}_{th}(D)$, $\frac{\lambda}{4} < D < \frac{\lambda}{2}$, $+jX_L$ inductor

$\tilde{Z}_{th}(D) = \tilde{Z}_{th}(D + \frac{\lambda}{2} n)$ for $n = 0, 1, 2 \ldots$

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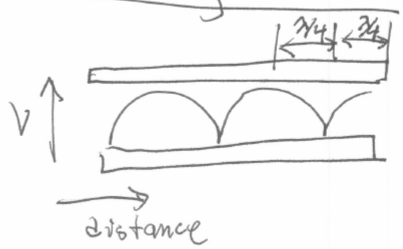
Standing Waves on a Transmission Line

Standing Waves



peaks and nulls of constructive interference

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Voltage Standing Wave Ratio

Voltage
Standing
Wave
Ratio

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

often reported in dB

$$20\log_{10}\frac{V_{max}}{V_{min}}$$

∞ for open circuit

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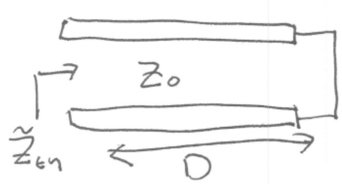
Total Voltage on a Shorted Line

Short Circuit



$$-\tilde{V}^+ = \tilde{V}^-$$

$$\tilde{V}(z) = j2\tilde{V}^+\left(\frac{\exp(-j\beta z) - \exp(+j\beta z)}{j2}\right)$$

$$= -j2\tilde{V}^+\sin\beta z$$

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Thevenin Equivalent Impedance of a Shorted Line

$$\tilde{i}(z) = 2\frac{\tilde{V}^+}{Z_0}\left(\frac{\exp(-j\beta z) + \exp(+j\beta z)}{2}\right)$$

$$= 2\frac{\tilde{V}^+}{Z_0}\cos\beta z$$

$$\tilde{Z}_{th}(D) = \frac{\tilde{V}(z)\big|_{z=-D}}{\tilde{i}(z)\big|_{z=-D}} = \boxed{j Z_0 \tan\beta D}$$

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