THT4: Arbitrary Loads on Transmission Lines

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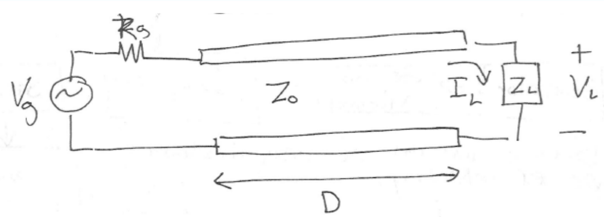
Review of Basic Time-Harmonic Solution



$$V(z) = V_1^+ \exp(-j\beta z) + V_1^- \exp(j\beta z)$$

$$i(z) = \frac{V_1^+}{Z_0} \exp(-j\beta z) - \frac{V_1^-}{Z_0} \exp(j\beta z)$$

$$V_1^+ = V_0^+ \exp(j\beta D) \qquad V_1^- = V_0^- \exp(-j\beta D)$$

$$V(z) = V_0^+ \exp(-j\beta[z-D]) + V_0^- \exp(j\beta[z-D])$$

Boundary Condition at Load Side

$$V(z) = V_0^+ \exp(-j\beta[z-D]) + V_0^- \exp(j\beta[z-D])$$

$$V_L = V(z)\big|_{z=D} = V_0^+ + V_0^-$$

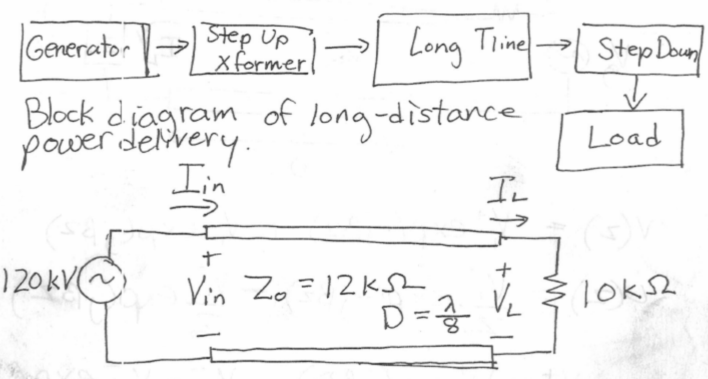
$$I_L = i(z)\big|_{z=D} = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_L}{Z_L}$$

$$\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^+}{Z_L} + \frac{V_0^-}{Z_L}$$

$$V_0^+ \underbrace{\left[\frac{1}{Z_0} - \frac{1}{Z_L}\right]}_{\frac{Z_L - Z_0}{Z_0 Z_L}} = V_0^- \underbrace{\left[\frac{1}{Z_L} + \frac{1}{Z_0}\right]}_{\frac{Z_L + Z_0}{Z_0 Z_L}}$$

High Voltage Line Example

Example: from Power



Step 1: Load Transformation

Completely Characterize $V(z)$, V_{in}, I_{in}, V_L, I_L for steady-state sinusoids

① Characterize Z_{in}

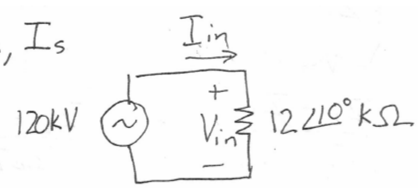
$$Z_{in} = Z_0 \left\{ \frac{Z_L + jZ_0 \tan\beta D}{Z_0 + jZ_L \tan\beta D} \right\} \qquad \beta D = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right) = \frac{\lambda}{4}$$

$$Z_{in} = 12{,}000 \left\{ \frac{10{,}000 + j12{,}000 \overbrace{\cancel{\tan\frac{\pi}{4}}}^{1}}{12{,}000 + j10{,}000 \overbrace{\cancel{\tan\frac{\pi}{4}}}^{1}} \right\}$$

$$= 12\angle 10^\circ \text{ k}\Omega \quad = \quad 11.8 + j2.2 \text{ k}\Omega$$

Step 2: Solve for Source-side Voltage & Current

② Find V_s, I_s



$$V_{in} = 120\angle 0^\circ \text{ kV}$$

$$I_{in} = \frac{V_{in}}{Z_{in}} = \frac{120\angle 0^\circ \text{ kV}}{12\angle 10^\circ \text{ k}\Omega} = 10\angle{-10^\circ} \text{ A}$$

Step 3: Enforce Source-side Continuity

③ Now solve for source-side boundary conditions.

$$V(z) = V_1^+ \exp(-j\beta z) + V_1^- \exp(+j\beta z)$$

$$V(0) = V_1^+ + V_1^- = 120\angle 0^\circ \text{ kV} \quad V_{in}$$

$$i(z) = \frac{V_1^+}{Z_0}\exp(-j\beta z) - \frac{V_1^-}{Z_0}\exp(+j\beta z)$$

$$i(0) = \frac{V_1^+}{Z_0} - \frac{V_1^-}{Z_0} = 10\angle{-10^\circ} \text{ A} \quad I_{in}$$

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Step 4: Solve for Forward/Backward Waves

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$$V_1^+ = \frac{V_{in} + Z_0 I_{in}}{2} \qquad V_1^- = \frac{V_{in} - Z_0 I_{in}}{2}$$

$$V_1^+ = \frac{120\angle 0^\circ \text{ kV} + (10\angle{-10^\circ})(10\text{ k}\Omega)}{2} =$$

$$= 119 - j11 \text{ kV} = 119.5\angle{-1^\circ} \text{ kV}$$

$$V_1^- = \frac{120\angle 0^\circ \text{ kV} - (10\angle{-10^\circ})(10\text{ k}\Omega)}{2}$$

$$= 1 + j11 \text{ kV} = 11\angle 85^\circ \text{ kV}$$

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Step 5: Solve for Load Side Voltage & Current

5 Find I_L, V_L

$$V_L = V(z)\big|_{z=D} = V_1^+ \exp(-j\beta D) + V_1^- \exp(j\beta D)$$

$$= 119.5\angle \underbrace{-1^\circ - 45^\circ}_{-46^\circ} + 11\angle \underbrace{85^\circ + 45^\circ}_{130^\circ} \text{ kV}$$

$$= (83.5 - j85.5) + (-7.0 + 8.5j) \text{ kV}$$

$$= 76.5 - j77 = 109\angle 45.2$$

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Step 5 Continued

$$I_L = i(z)\big|_{z=D} = \frac{V_1^+}{Z_0}\exp(-j\beta D) - \frac{V_1^-}{Z_0}\exp(j\beta D)$$

$$= \frac{119.5\angle -46^\circ \text{ kV}}{12 \text{k}\Omega} - \frac{11\angle 130^\circ \text{ kV}}{12 \text{k}\Omega}$$

$$= (7.0 - j7.1) - (-0.6 + 0.7j)$$

$$= 7.6 - j7.8 = 10.9\angle 45.7^\circ$$

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Step 6: Check Load Side Answer with Ohm's Law

6) Check What does $\frac{V_L}{I_L} = ?$

$$\frac{V_L}{I_L} = \frac{109\angle 45.2^\circ \text{ kV}}{10.9\angle 45.7^\circ} = 10\angle -0.5^\circ \text{ k}\Omega$$

the original Load