THT4: Arbitrary Loads on Transmission Lines

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Review of Basic Time-Harmonic Solution

\[ V(z) = V_1^+ \exp(-j\beta z) + V_1^- \exp(j\beta z) \]
\[ I(z) = \frac{V_1^+}{Z_0} \exp(-j\beta z) - \frac{V_1^-}{Z_0} \exp(j\beta z) \]
\[ V_i^+ = V_0^+ \exp(j\beta D) \quad V_i^- = V_0^- \exp(j\beta D) \]
\[ V(z) = V_0^+ \exp(j\beta [z-D]) + V_0^- \exp(j\beta [z-D]) \]
Boundary Condition at Load Side

\[ V(z) = V_0^+ \exp(j \beta[z - D]) + V_0^- \exp(j \beta[z - D]) \]

\[ V_L = \left. V(z) \right|_{z = D} = V_0^+ + V_0^- \]

\[ I_L = \left. i(z) \right|_{z = D} = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^+ - V_0^-}{Z_L} \]

\[ \frac{V_0^+}{Z_o} - \frac{V_0^-}{Z_o} = \frac{V_0^+}{Z_L} + \frac{V_0^-}{Z_L} \]

\[ V_0^+ \left[ \frac{1}{Z_L} - \frac{1}{Z_o} \right] = V_0^- \left[ \frac{1}{Z_L} + \frac{1}{Z_o} \right] \]

\[ \frac{Z_L - Z_o}{Z_o Z_L} \quad \frac{Z_o + Z_L}{Z_o Z_L} \]

High Voltage Line Example

Example: From Power

Block diagram of long-distance power delivery.

\[ V_{in} \quad Z_o = 12 \, \text{k} \Omega \]

\[ D = \frac{1}{2} \quad V_L \quad 10 \, \text{k} \Omega \]
Step 1: Load Transformation

Completely Characterize $V(Z), V_{in}, I_{in}, V_{o}, I_{o}$ for steady-state sinusoids

1. Characterize $Z_{in}$

$$Z_{in} = Z_0 \left( \frac{Z_0 + jZ_0 \tan \beta D}{Z_0 + j Z_0 \tan \beta D} \right)$$

$$Z_{in} = 12,000 \left( \frac{10,000 + j12,000 \tan \frac{\pi}{4}}{12,000 + j10,000 \tan \frac{\pi}{4}} \right)$$

$$= 12,210^\circ \ \Omega = 11.8 + 1.2 \ \Omega$$

Step 2: Solve for Source-side Voltage & Current

2. Find $V_s, I_s$

$$V_{in} = 120^\circ \ \text{kV}$$

$$I_{in} = \frac{V_{in}}{I_{in}} = \frac{120^\circ \ \text{kV}}{12210^\circ \ \Omega} = 104^\circ \ \text{A}$$
Step 3: Enforce Source-side Continuity

Now, solve for source-side boundary conditions.

\[ V(z) = V_i^+ \exp(-j\beta z) + V_i^- \exp(j\beta z) \]

\[ V(0) = V_i^+ + V_i^- = 120 \angle 2^\circ \text{ kv} - V_{in} \]

\[ I(z) = \frac{V_i^+}{Z_0} \exp(-j\beta z) - \frac{V_i^-}{Z_0} \exp(j\beta z) \]

\[ I(0) = \frac{V_i^+}{Z_0} - \frac{V_i^-}{Z_0} = 10 \angle 10^\circ \text{ a} \ I_{in} \]

Step 4: Solve for Forward/Backward Waves

\[ V_i^+ = \frac{V_{in} + Z_0 I_{in}}{Z_0} \]

\[ V_i^- = \frac{V_{in} - Z_0 I_{in}}{Z_0} \]

\[ V_i^+ = \frac{120 \angle 2^\circ \text{ kv} + (10 \angle 10^\circ)(10 \text{ k}\Omega)}{2} = 119 - j11 \text{ kv} = 119.5 \angle -1^\circ \text{ kv} \]

\[ V_i^- = \frac{120 \angle 2^\circ \text{ kv} - (10 \angle 10^\circ)(10 \text{ k}\Omega)}{2} = 1 + j11 \text{ kv} = 11 \angle 85^\circ \text{ kv} \]
Step 5: Solve for Load Side Voltage & Current

5. Find \( I_L, V_L \)

\[
V_L = V(\bar{z}) \bigg|_{\bar{z} = D} = V_i^+ \exp(-jBD) + V_i^- \exp(jBD)
\]

\[
= 119.5 \angle -10° - 45° + 11 \angle 285° + 45° \text{ kV}
\]

\[
= (83.5 - j85.5) + (-7.0 + 8.5j) \text{ kV}
\]

\[
= 76.5 - j77. \quad = 109 \angle 45°
\]

Step 5 Continued

\[
I_L = \bar{I}(\bar{z}) \bigg|_{\bar{z} = D} = \frac{V_i^+}{Z_o} \exp(-jBD) + \frac{V_i^-}{Z_o} \exp(jBD)
\]

\[
= \frac{119.5}{12kΩ} \frac{-46°}{12kΩ} \text{ kV} + \frac{11130°}{12kΩ} \text{ kV}
\]

\[
= (7.0 - j7.1) - ( -0.6 + 0.7j)
\]

\[
= 7.6 - j7.8 \quad = 10.9 \angle 45.7°
\]
Step 6: Check Load Side Answer with Ohm's Law

6) Check what does $\frac{V_L}{I_L} = ?$

$$\frac{V_L}{I_L} = \frac{109.45 \angle -45^\circ}{10.9 \angle 45^\circ} = 10 \angle -90^\circ \text{ k}\Omega$$

the original Load