Why Are Lines Lossy?

Teaser Question: When was the first undersea telegraph installed?

For lossy lines, either $R$ or $G$ is non-zero.

Notes:
1) We usually study lossy line effects for sinusoidal excitation since loss is not as important for DC switching and logic pulses. These traces are usually too short to notice loss. This is changing, however, as frequencies continue to climb.
Mathematical Solution for Lossy Line

2) For sinusoidal excitation, solution is
\[ V(z) = V_0^+ \exp(-\gamma z) + V_0^- \exp(+\gamma z) \]
\[ i(z) = \frac{V_0^+}{Z_0} \exp(-\gamma z) - \frac{V_0^-}{Z_0} \exp(+\gamma z) \]
\[ Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \text{ now imaginary} \]
\[ \gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} \]

3) What causes loss?
- Ohmic resistance, from metal
- Conduction in separating medium
- Radiation effects

Time-Domain Solution

Look carefully:
Let us say we had a forward prop. wave
\[ V(z) = V_0^+ \exp(-\gamma z) \]
\[ = V_0^+ \exp(-[\alpha + j\beta]z) \]
\[ = V_0^+ \exp(-\alpha z) \exp(-j\beta z) \]
\[ V(t, z) = \mathcal{F}_t V(z) \exp(j\omega t) \]
\[ = V_0^+ \exp(-\alpha z) \cos(\omega t - \beta z) \]
**Magnitude of Voltage over Distance**

Envelope

\[ |V(z)| \]

\[ z = \begin{array}{c}
0 \\
D 
\end{array} \]

\[ \alpha \text{ has units of Napiers per meter} \]

More useful value is dB/m

\[ P(z) = \frac{|V_0|^2}{2Z_0} \cdot | \exp(-kx) \exp(-j \beta z) |^2 \]

\[ = \frac{|V_0|^2}{2Z_0} \exp(-Z \alpha z) \]

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**Attenuation Coefficients**

\[ P(z+lm) = \frac{|V_0|^2}{2Z_0} \exp(-2\alpha z) \exp(-2\alpha \cdot lm) \]

\[ \text{dB Loss/m} = 10 \log_{10} \frac{P(z)}{P(z+lm)} \]

\[ = 10 \log_{10} \exp(-2\alpha) = 10 \log_{10} 10^{-2\alpha} \log_{10} e \]

\[ = -20 \alpha \log_{10} e \]

\[ \text{Loss} = 8.7\alpha \text{ dB/m} \]
**Example: Converting a Spec**

Example

<table>
<thead>
<tr>
<th>Loss @ 1.0 GHz</th>
<th>Loss @ 2.0 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coaxial Cable</td>
<td></td>
</tr>
<tr>
<td>1.6 dB/m</td>
<td>3.0 dB/m</td>
</tr>
<tr>
<td>( \alpha = 0.184 \text{ Np/m} )</td>
<td>( \alpha = 0.345 \text{ Np/m} )</td>
</tr>
</tbody>
</table>

**Low Loss Transmission Line Formulas**

Low Loss Lines

\[
\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}
\]

\[
= j\omega \sqrt{LC} \sqrt{(1 + \frac{R}{j\omega L})(1 + j\omega C)}
\]

\[
= j\omega \sqrt{LC} \left[ 1 + \frac{R}{j\omega L} \right] \left[ 1 + \frac{G}{j\omega C} \right]
\]
Key Low-Loss Results

Further approximation:

\[ Y \approx j\omega \sqrt{LC} \left( 1 + \frac{1}{j\omega^2} \left[ \frac{R}{L} + \frac{G}{C} \right] \right) \]

then

\[ \beta \approx \omega \sqrt{LC} \quad \alpha \approx \frac{R}{2\sqrt{L}} + \frac{G}{2\sqrt{C}} \]

\[ Z_0 \approx \sqrt{\frac{L}{C}} \]

Lossy Line Load Transformation

\[ Z_{in} = Z_0 \left[ \frac{Z_L + Z_0 \tanh(Y_L)}{Z_0 + Z_L \tanh(Y_L)} \right] \]

Hyperbolic vs regular tangent:

\[ \tan x = \frac{\exp(jx) - \exp(-jx)}{j [\exp(jx) + \exp(-jx)]} \]

\[ \tanh j\theta D = j \tan \theta D \]
**Frequency-Dependent Attenuation**

\[
\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} \\
\alpha_c: \text{Conductor losses} \quad \alpha_d: \text{Dielectric losses}
\]

\[
\alpha_c = \frac{R}{2Z_0} \quad \alpha_d = \frac{GZ_o}{2}
\]

Starts out small for low frequencies and creeps upward with increasing frequency. Eventually, this term dominates.

**Loss on a Coaxial Cable**

**Skin Effect for High Frequencies**

Microstrip cross section Coaxial X-section

\[
\alpha_c = \frac{\sqrt{\mu}}{4Z_0} \sqrt{\frac{M}{\pi \sigma}} \left[\frac{1}{a} + \frac{1}{b}\right] \\
\sigma \rightarrow \text{conductivity}
\]
Example: Cable Modem ISP

Cable Modem: FDM

TV cable specs
Cu: 5.96 x 10^-7 Ω^-1 m^-1
α = 0.5 mm
b = 5 mm
Z_o = 75 Ω

Solution

What is the loss in dB for cables carrying:
① the lowest uplink frequency? (5 MHz)
② the highest downlink frequency? (1 GHz)

\[ \alpha (f) = \alpha_c \]
\[ = \frac{\frac{1}{4\pi} \sqrt{\frac{\mu}{\sigma}}}{\frac{1}{4\pi} \sqrt{\frac{\mu}{\sigma}}} \]
\[ = \frac{1.3 \times 10^{-3} \text{ Np/m}}{0.012 \text{ dB/m}} \]
\[ \alpha (5 \text{ MHz}) = 0.019 \text{ Np/m} = 0.165 \text{ dB/m} \]

\[ \alpha (1 \text{ GHz}) = 1.3 \times 10^{-3} \text{ Np/m} \]