1. Hall Sensor

(a) see diagram below

(b) see diagram below

(c) If positive charges carried electrical current, the direction of current flow or direction of voltage drop across the Hall sensor would not change; however, the gradient of current will be reversed in the x-direction.

(d) To find total current, $I$, we must integrate current density across the cross-sectional area of flow:

$$I = \int_{0}^{w} \vec{K} \cdot d\vec{x} = \int_{0}^{w} K_0 \left(1 - \frac{x'}{w}\right) \cdot dx' = K_0 \left[ x' - \frac{x'^2}{2w} \right]_{0}^{w} = \frac{K_0 w}{2}$$

(e) For the DC resistance in the absence of a magnetic field, the equation for resistance is

$$R = \frac{L}{\sigma A} = \frac{L}{\sigma \delta w}$$

Some students assumed $L$ to be infinite from part (f), resulting in infinite resistance; this answer was accepted also.

(f) Magnetic field on the $xy$-plane is given by

$$\vec{H}(x, y, 0) = \int_{S} \frac{\vec{K} (\vec{r}) \times (\vec{r} - \vec{r'}) dS'}{4\pi ||\vec{r} - \vec{r'||}^3}$$

$$= \int_{0}^{w} dx' \int_{-\infty}^{+\infty} dy' \frac{K_0 \left(1 - \frac{x'}{w}\right) \hat{y} \times [(x - x')\hat{x} + (y - y')\hat{y}]}{4\pi \left[(x - x')^2 + (y - y')^2\right]^{\frac{3}{2}}}$$
\[
= -\frac{K_0 \hat{z}}{4\pi} \int_0^w \int_{-\infty}^{+\infty} \frac{1 - \frac{x'}{w}}{(x - x')^2 + (y - y')^2} \, dy' \, dx'
\]

2. Inductive Power Transfer

- The frequency of the AC current, \( I \), is increased.

- The reader coil is encased in a high-dielectric plastic.

- The area of the charging device’s collecting coil increases.

- A matching capacitor is added in series with the exciter loop.

+ An inductor is added in series with the charging circuitry, \( Z_L \).

- The amplitude of the current in the exciter coil is increased.

+ The charging device’s collecting coil is kept parallel and moved several inches above the mat.

+ The charging device’s collecting coil is tilted perpendicular to the mat.

= The system is placed in a vacuum chamber and all of the air is pumped out.

+ A metal foil is inserted between the exciter and collector coils.

Note that some students interpreted the question with “reverse polarity” (i.e. + means more power transfer/less charging time, - means less power/more charging time). These students were docked 2 points for not following instructions and their answers were then flipped for grading purposes. Everyone was given the benefit of the doubt in the sense that each test was graded both ways (proper polarity and flipped polarity) and the higher of the two grades was granted.