

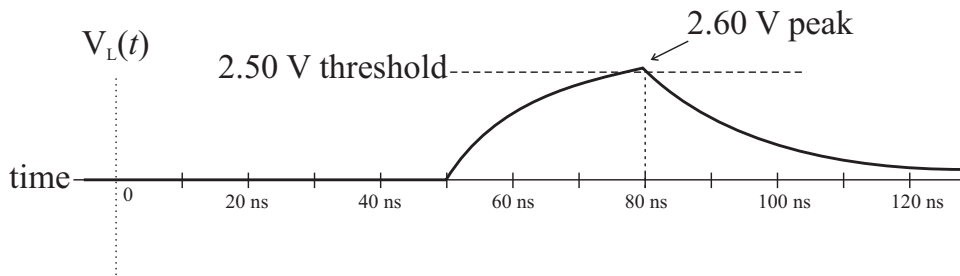
ECE 3025 Homework 4: Reactive and Nonlinear Loads

Solutions

1. The total resistance as seen by the capacitor is $(2000\Omega) \parallel (100\Omega) = 95\Omega$ and the subsequent time constant for this circuit is $\tau = 38$ ns. Following directly from the notes, we can sketch out the following solution:

$$V_L(t) = \begin{cases} 0 & t < 50 \text{ ns} \\ 4.76 \left[1 - \exp\left(-\frac{t-5 \times 10^{-8}}{3.8 \times 10^{-8}}\right) \right] & 50 \text{ ns} \leq t < 80 \text{ ns} \\ 4.76 \left[\exp\left(-\frac{t-8 \times 10^{-8}}{3.8 \times 10^{-8}}\right) - \exp\left(-\frac{t-5 \times 10^{-8}}{3.8 \times 10^{-8}}\right) \right] & t \geq 80 \text{ ns} \end{cases}$$

A sketch of this function is shown below:



Note that the last term in the solutions (for $t > 80$ ns) can take the following form after several exponential identities are applied:

$$\begin{aligned} 4.76 \left[\exp\left(-\frac{t-8 \times 10^{-8}}{3.8 \times 10^{-8}}\right) - \exp\left(-\frac{t-5 \times 10^{-8}}{3.8 \times 10^{-8}}\right) \right] &= \\ 5.72 \exp\left(-\frac{t-5 \times 10^{-8}}{3.8 \times 10^{-8}}\right) &= 2.60 \exp\left(-\frac{t-8 \times 10^{-8}}{3.8 \times 10^{-8}}\right) \end{aligned}$$

So the answer may depend on how the student reduces the exponentials.

For part (b), the peak of the output is at 2.60 – barely enough to cross the threshold logic level of 2.5 V specified in the problem. This point is crossed at $t = 78$ ns.

Note: With the switch up, the source side is not matched to the line. Thus, there will be some diminishing copies of the distorted pulse occurring every odd interval of T in time. The students are not responsible for sketching or solving these later reflections.

2. (a) We use this basic iterative equation to calculate the solution for this problem:

$$I_L^{\text{new}} = \frac{2}{Z_0} V^+ - \frac{1}{Z_0} f(I_L^{\text{old}})$$

where

$$V_L = f(I_L) = V_0 \ln \left(\frac{I_L}{I_0} + 1 \right)$$

and $V_0 = 0.4$ V and $I_0 = 0.5$ mA. Fortunately, this equation converges very rapidly for a diode characteristic. For the 2V forward traveling wave, the answers are $V_L = 1.56$ V and $I_L = 24.4$ mA.

- (b) Steady-state forward and reflected voltages:

$$V^+ = \frac{V_L + I_L Z_0}{2} = 2.00 \text{ V}$$

$$V^- = \frac{V_L - I_L Z_0}{2} = -0.44 \text{ V}$$

- (c) If the device is flipped in the circuit, we have to modify the characteristic as follows:

$$V_L = f(I_L) = -V_0 \ln \left(\frac{-I_L}{I_0} + 1 \right)$$

We'll use the alternative iterative equation for this example, since the other one does not converge nicely:

$$V_L^{\text{new}} = 2V^+ - Z_0 f^{-1}(V_L^{\text{old}})$$

The answers are $V_L = 3.95$ V and $I_L = 0.5$ mA.

- (d) Steady-state forward and reflected voltages:

$$V^+ = \frac{V_L + I_L Z_0}{2} = 2.00 \text{ V}$$

$$V^- = \frac{V_L - I_L Z_0}{2} = 1.95 \text{ V}$$

3. Chaotic oscillator: There may be several valid answers for this circuit. From the bifurcation map shown in class, one can see that $R_s = -24.35\Omega$ is one point that may slip into 3-state oscillations. This can be simulated with the online code using the following syntax (transit time 1 ns, $Z_0 = 50\Omega$, $V_s = 0.2$, $R_s = -24.35$):

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>> NonlinearTlinePlot( 1e-9, 50, .2, -24.35 );
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Note that the transit time is not given in the problem, but does not change the oscillation values. The output of the transmission line is

Output of Transmission Line with $R_s = -24 \Omega$

