ECE 3025 Homework 5: Sinusoids on Transmission Lines

Solutions

1. The load impedance – a capacitor in parallel with a resistor – as a function of frequency is given by the following expression:

$$Z_L = \frac{200}{1 + j400\pi f C}$$

where C is 10 pF. The argument βD can be written as

$$\beta D = \left(\frac{\omega}{v_p}\right) D = \left(\frac{2\pi f}{D/T}\right) D = 2\pi f T$$

With these two expressions, we now have enough to evaluate the general expression for equivalent input impedance:

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(2\pi f T)}{Z_0 + jZ_L \tan(2\pi f T)}\right]$$

Below is a table of impedance, Z_{in}, for each frequency:

f	βD	Z_L (Ω)	Z_{in} (Ω)
0 MHz	0	200	200
100 MHz	π	77.5-j97.4	77.5-j97.4
300 MHz	3π	13.1-j49.6	13.1-j49.6
500 MHz	5π	4.9-j31.0	4.9-j31.0

2. If the load is purely reactive, we can write the load resistance in the form $Z_L = jX$, where X is a real constant (positive for inductors, negative for capacitors). Plugging this value into our formula for reflection coefficient gives

$$\begin{aligned}
|\Gamma_L| &= \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| \\
&= \left|\frac{jX - Z_0}{jX + Z_0}\right| \\
&= \left|\frac{-\sqrt{X^2 + Z_0^2}\exp\left(-j\tan^{-1}\frac{X}{Z_0}\right)}{\sqrt{X^2 + Z_0^2}\exp\left(j\tan^{-1}\frac{X}{Z_0}\right)}\right| \\
&= \frac{\left|-\exp\left(-j\tan^{-1}\frac{X}{Z_0}\right)\right|}{\left|\exp\left(j\tan^{-1}\frac{X}{Z_0}\right)\right|} \\
&= 1
\end{aligned}$$

Since VSWR$= \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$, the voltage standing wave ratio is always infinity for purely reactive loads.

3. Refer to Figure 1 for a simplified circuit diagram of the damaged line. Here the 75Ω load, since it is still matched to the last 6.73m of cable, is transformed without change to the point of damage. Now we need to calculate the equivalent load at this point – a 118Ω resistance in parallel with a capacitor and the 75Ω resistance – in order to transform the impedance further up the line:

$$Z_{eq} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta D}{Z_0 + jZ_L \tan \beta D} \right] \qquad Z_L = \frac{R_D(Z_0 - j\frac{1}{2\pi f C_D})}{R_D + Z_0 - j\frac{1}{2\pi f C_D}}$$

where R_D and C_D are the damage area resistance and capacitance, respectively. Once Z_{eq} is known, the voltage and current at the front of the line are given by the voltage-divider equation:

$$V_S = V_{source} \frac{Z_{eq}}{Z_0 + Z_{eq}} \qquad I_S = \frac{V_{source}}{Z_0 + Z_{eq}}$$

The general voltage-current relationships on the transmission line are given by the following system of equations:

$$\begin{aligned} \tilde{v}(z) &= V^+ \exp(-j\beta z) + V^- \exp(j\beta z) \\ \tilde{i}(z) &= \frac{V^+}{Z_0} \exp(-j\beta z) - \frac{V^-}{Z_0} \exp(j\beta z) \end{aligned}$$

We can always solve for V^+ and V^- by setting $\tilde{v}(0)$ and $\tilde{i}(0)$ equal to V_S and I_S respectively. This results in the following relationships:

$$V^+ = \frac{V_S + I_S Z_0}{2} \qquad V^- = \frac{V_S - I_S Z_0}{2}$$

which may be plugged back into the transmission line equation to solve for voltage and current *at the end of the line.* Whatever current results (V_L and I_L) then divides across the load resistors.

Since the problem requests the computation to be made at several frequencies, it is probably easiest to enter the equations into a computer script using a software like MatLab™ and re-run the code with different values of frequency. Assuming that the source voltage has amplitude 1V (RMS), here is the power that is absorbed by the television load for each case:

Case	Load at D Z_L (Ω)	Power to Load (mW)	Loss (dB)
Undamaged	75	3.3	–
100 MHz	$75.1 - j35.4$	0.7	6.8
400 MHz	$48.8 - j14.3$	1.7	2.8
900 MHz	$46.5 - j6.6$	1.9	2.5

Note that the loss is frequency dependent. This is interesting for the application of cable television signals, since analog television signals are frequency-division multiplexed across the spectrum between DC and 1 GHz. Thus, the damage will not affect all channels evenly.

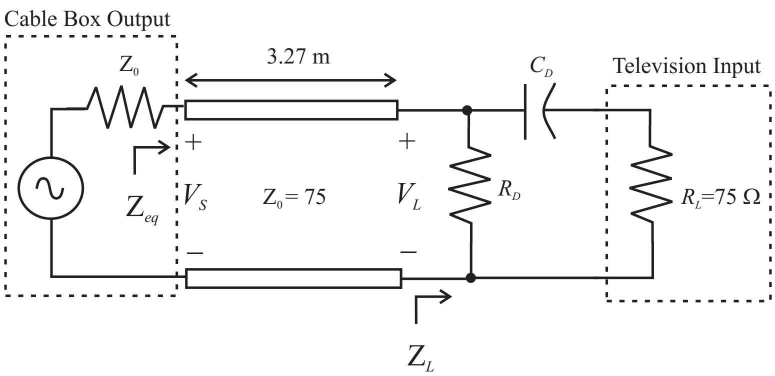

Figure 1: Equivalent circuit diagram for Problem 3.