1. Setting up E-Field Integrals:

(a) \[ \mathbf{E}(\mathbf{r}) = \int_{\mathcal{S}} \frac{\rho_s (\mathbf{r} - \mathbf{r}')} {4\pi \epsilon ||\mathbf{r} - \mathbf{r}'||^3} \mathbf{d}S \]

\[ \mathbf{E}(x\hat{x} + y\hat{y} + z\hat{z}) = \int_0^R 2\pi \frac{\rho_s (x\hat{x} + y\hat{y} + z\hat{z} - \rho \cos \phi \hat{x} - \rho \sin \phi \hat{y} - \mathbf{0}) \rho d\rho d\phi} {4\pi \epsilon ||x\hat{x} + y\hat{y} + z\hat{z} - \rho \cos \phi \hat{x} - \rho \sin \phi \hat{y} - \mathbf{0}||^3} \]

\[ \mathbf{E}(x, y, z) = \frac{\rho_s}{4\pi \epsilon} \int_0^R 2\pi \frac{[(x - \rho \cos \phi)\hat{x} + (y - \rho \sin \phi)\hat{y} + z\hat{z}] \rho d\rho d\phi} {||[(x - \rho \cos \phi)^2 + (y - \rho \sin \phi)^2 + z^2]||^2} \]

(b) \[ \mathbf{E}(\mathbf{r}) = \int_{\mathcal{S}} \frac{\rho_0 \sqrt{1 - (z')^2}} {4\pi \epsilon ||\mathbf{r} - \mathbf{r}'||^3} r^2 \sin \theta d\theta d\phi \]

\[ \mathbf{E}(x, y, z) = \frac{\rho_0}{4\pi \epsilon} \int_0^{2\pi} \frac{\sqrt{1 - (z')^2} [(x - \sin \theta \cos \phi)\hat{x} + (y - \sin \theta \sin \phi)\hat{y} + (z - \cos \theta)\hat{z}] r^2 \sin \theta d\theta d\phi} {||[(x - \sin \theta \cos \phi)^2 + (y - \sin \theta \sin \phi)^2 + (z - \cos \theta)^2]||^2} \]

\[ = \frac{\rho_0}{4\pi \epsilon} \int_0^{2\pi} \frac{[(x - \sin \theta \cos \phi)\hat{x} + (y - \sin \theta \sin \phi)\hat{y} + (z - \cos \theta)\hat{z}] \sin^2 \theta d\theta d\phi} {||[(x - \sin \theta \cos \phi)^2 + (y - \sin \theta \sin \phi)^2 + (z - \cos \theta)^2]||^2} \]

(c) Since this line charge spirals in space and is not aligned along any particular axis, it is easiest to integrate the curve over the parameter \( t \). What we must realize first, however, is that \( dL = \sqrt{R'^2 + 1} dt \). In other words, for every 1 unit of \( t \) advanced, we advance \( \sqrt{R'^2 + 1} \) units of physical distance along the helix.
\[
\vec{E}(\vec{r}) = \int_{-\infty}^{+\infty} \frac{\rho_L(t')(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \frac{\sqrt{R^2 + 1} dt}{4\pi\epsilon}
\]

\[
\vec{E}(x, y, z) = \frac{\rho_L(\vec{r})}{4\pi\epsilon} \int_{-\infty}^{+\infty} \frac{(x \hat{x} + y \hat{y} + z \hat{z} - x'(t) \hat{x} - y'(t) \hat{y} - z'(t) \hat{z}) dt}{|x \hat{x} + y \hat{y} + z \hat{z} - x'(t) \hat{x} - y'(t) \hat{y} - z'(t) \hat{z}|^3}
\]

\[
\vec{E}(x, y, z) = \frac{\rho_L \sqrt{R^2 + 1}}{4\pi\epsilon} \int_{-\infty}^{+\infty} \frac{[(x - R \cos t) \hat{x} + (y - R \sin t) \hat{y} + (z - t) \hat{z}] dt}{[(x - R \cos t)^2 + (y - R \sin t)^2 + (z - t)^2]^\frac{3}{2}}
\]

As a side note, if you let \( R \to 0 \), this last integral should become the solution to the infinite line charge problem discussed in class.

2. For a point of observation \( P = (8, 12, 2) \) and a point of charge \( A = (4, 3, 5) \):

\[
\vec{E} = \frac{Q(\vec{P} - \vec{A})}{4\pi\epsilon_0|\vec{P} - \vec{A}|^3} = 16.5(4\hat{x} + 9\hat{y} - 3\hat{z}) \text{ Volts/m}
\]

The point of observation (8,12,2) has the following cylindrical coordinate parameters:

\( \rho = 14.4 \quad \phi = 0.9828 \quad R = 2.0 \)

When plugged into the conversion formulas, the field at this point is

\[
\vec{E} = 160.2\hat{\rho} + 27.5\hat{\phi} - 49.5\hat{z} \text{ Volts/m}
\]

These conversion problems can get kind of complicated, but a quick sanity check is to take the norm of \( \vec{E} \) in both Cartesian and Cylindrical or Spherical coordinates; the magnitudes should be identical in any coordinate system.

3. The total charge, \( Q \), is equal to

\[
Q = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_v(x, y, z) \, dx \, dy \, dz
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-|x| - |y| - |z|) \, dx \, dy \, dz
\]

\[
= 2 \int_{0}^{\infty} \exp(-x) \, dx \int_{0}^{\infty} \exp(-y) \, dy \int_{0}^{\infty} \exp(-z) \, dz
\]

\[
= 8 \text{ C}
\]
4. We know how to solve for the line charge if it rests on the z-axis:

\[ \vec{E} = \frac{\rho L}{2\pi\epsilon_0 (x^2 + y^2)} [x\hat{x} + y\hat{y}] \]

This is a basic adaptation from the formula in the book. If the line of charge is shifted to \( y = y_0 \), then the modified expression becomes:

\[ \vec{E} = \frac{\rho L}{2\pi\epsilon_0 (x^2 + (y - y_0)^2)} [x\hat{x} + (y - y_0)\hat{y}] \]

Thus, total field for the two lines in this problem is given by

\[ \vec{E} = \frac{\rho L}{2\pi\epsilon_0 (x^2 + (y - y_0)^2)} [x\hat{x} + (y - y_0)\hat{y}] \]

For the first observation point in part (a), \((x, 0, z)\), this expression becomes

\[ \vec{E} = \frac{\rho L y_0}{\pi\epsilon_0 (x^2 + y_0^2)} \hat{y} = -\frac{8632}{x^2 + 0.36} \hat{y} \text{ Volts/m} \]

For the second observation point in part (b), \((2, 3, 4)\), this expression becomes

\[ \vec{E} = \frac{7193}{2^2 + 2.4^2} [2\hat{x} + 2.4\hat{y}] - \frac{7193}{2^2 + 3.6^2} [2\hat{x} + 3.6\hat{y}] = 626\hat{x} + 242\hat{y} \text{ Volts/m} \]

Note that there is no dependence on \( z \) in this field.

5. From the previous problem, we know that the E-field from the first line of charge is going to be

\[ \vec{E} = \frac{\rho L}{2\pi\epsilon_0 (x^2 + (y - y_0)^2)} [x\hat{x} + (y - y_0)\hat{y}] = \frac{1349}{x^2 + (y - 0.4)^2} [x\hat{x} + (y - 0.4)\hat{y}] \]

The point of observation is the other line, which can be represented as \((0, -0.4, z)\), and leads to a field of:

\[ \vec{E} = \frac{1349}{0^2 + (-0.4 - 0.4)^2} [0\hat{x} + (-0.4 - 0.4)\hat{y}] = -1686\hat{y} \text{ Volts/m} \]

To get force per unit meter that line 1 exerts on line 2, we multiply this field by the charge density of the other line:

\[ \vec{F} = \rho_L \vec{E} = -1.64 \times 10^{-4} \hat{y} \text{ Newtons/m} \]

The force that line 2 exerts on line 1 would be equal and opposite: \(+1.64 \times 10^{-4} \hat{y} \text{ Newtons/m} \).