Name: __________________________
GTID: __________________________

ECE 3025: Electromagnetics
TEST 1 (Spring 2008)

• Please read all instructions before continuing with the test.

• This is a closed notes, closed book, closed calculator, closed friend, open mind test. You should only have writing instruments on your desk when you take this test. If I find anything on your desk (excluding the test itself, writing instruments, and life-or-death medication) I will turn you in for an honor code violation. I am serious.

• Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.

• Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.

• All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.

• You have 80 minutes to complete this examination. When I announce a “last call” for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.

• I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.

• Have a nice day!

Pledge Signature: __________________________

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.
(1) **RFID on Metallic Surfaces:** Below is a picture of an Impinj^\text{TM}^ 915 MHz inductive RFID tag (left), consisting of an radio-frequency integrated circuit (RFIC) in the center connected to a small loop antenna. This is usually fabricated atop a piece of insulating plastic. One way to model this system is shown below (right) where the antenna sends a forward-propagating voltage down a coplanar strip transmission line terminated by the RFIC. The RFIC can modulate information on the reflected sinusoidal wave by switching between different loads \( Z_A \) and \( Z_B \).

![Diagram of RFID tag](image)

When the RFID tag is placed on a metal surface, the topology of the circuit changes dramatically. One way to model this new topology is with a *microstrip* transmission line; the antenna and feed traces now beat against the metal surface as if it were a ground plane underneath the plastic/dielectric, just like a printed circuit board!

![Diagram of RFID on metal](image)

**Answer the following questions based on this scenario. (36 points)**

(a) For the free-space RFID tag, if the chip is excited by the antenna with a forward propagating wave of 1V, what is the amplitude and phase of the reflected signals if \( Z_A \) is an open circuit and \( Z_B \) is a perfect short? (5 points)
(b) For the free-space RFID tag, what is the VSWR (in dB) for either transmission state in (a)? (5 points)

(c) When the RFID tag is brought closer and closer to metal, explain how $Z_0$ changes in the second circuit model. (10 points)

(d) What is the mathematical relationship between $V^+$ and $V^-$ (in terms of $\beta$, $D$, and $Z_0$) on the transmission line in the near-metal circuit model? (Hint: What does Kirchoff’s Current Law state about the relationship between $I_L$ and $I_S$?) (16 points)

(e) (Bonus +5 Points): What is the Thevenin equivalent of the antenna in this circuit as seen from the terminals of the RFIC in terms of $\beta$, $D$, and $Z_0$?
(2) **PCB Reflections:** Below is an overhead view of microstrip traces linking the output of chip 1 with two other chips on a high-speed digital printed circuit board. The transit time is labeled on each leg of transmission line (a dielectric slab and ground plane are beneath the traces, of course). All sources, loads, and fan-outs are mismatched. A very short pulse (treat as an impulse) is sent to the output of chip 1 at \( t = 0 \). List, in order, the first ten unique times \( t \) that the input would likely be non-zero at chip 2. (20 points)

![Diagram of PCB with microstrip traces and transit times: \( T_1 = 4 \text{ ns}, T_2 = 2 \text{ ns}, T_3 = 3 \text{ ns} \).]
(3) **The Death Star:** The circuit below represents a high-speed digital interconnect that is switched according to the following states:
- State 0: Both switches are open and both lines are uncharged.
- State 1: Immediately after switch A is closed onto the DC source.
- State 2: Switch A has been closed for a while.
- State 3: Immediately after switch B is closed.
- State 4: Switch B has been closed for a while.
- State 5: Immediately after Switch B re-opens.

Fill out the following table according to these switching states. Unsimplified fractional answers are OK. Assume all backwards propagating waves are measured from the right-most side of the transmission line. Assume all forward propagating waves are measured from the left-most side of the transmission line. (44 points):

<table>
<thead>
<tr>
<th></th>
<th>$V_1$</th>
<th>$V_X$</th>
<th>$V_Y$</th>
<th>$V_1^+$</th>
<th>$V_1^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>State 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>State 2</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>State 3</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>State 4</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>State 5</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Formula Sheet

\[
\frac{\partial^2 v(z,t)}{\partial z^2} = LC \frac{\partial^2 v(z,t)}{\partial t^2} \quad \frac{\partial^2 i(z,t)}{\partial z^2} = LC \frac{\partial^2 i(z,t)}{\partial t^2} \quad Z_0 = \sqrt{\frac{L}{C}} \quad \nu_p = \frac{1}{\sqrt{LC}}
\]

\[
\lambda f = \nu_p \quad \omega = \frac{2\pi f}{\lambda} \quad \beta = \frac{2\pi}{\lambda} \quad \text{Reflection: } \Gamma_{L,G} = \frac{Z_{L,G} - Z_0}{Z_{L,G} + Z_0} \quad \text{Transmission: } 1 + \Gamma_{L,G}
\]

Phasor Transform: \( A \cos(2\pi ft + \phi) \rightarrow A \exp(j\phi) \)

Reverse Transform: \( \tilde{x} \rightarrow \text{Real} \{ \tilde{x} \exp(j2\pi ft) \} \)

\[
v(z,t) = V^+ f(t - \frac{z}{\nu_p}) + V^- g(t + \frac{z}{\nu_p}) \quad \tilde{v}(z) = V^+ \exp(-\gamma z) + V^- \exp(+\gamma z)
\]

\[
i(z,t) = \frac{V^+}{Z_0} f(t - \frac{z}{\nu_p}) - \frac{V^-}{Z_0} g(t + \frac{z}{\nu_p}) \quad \tilde{i}(z) = \frac{V^+}{Z_0} \exp(-\gamma z) - \frac{V^-}{Z_0} \exp(+\gamma z)
\]

\[
\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad Z_{in} = Z_0 \left( Z_L + jZ_0 \tan \beta D \right)
\]

\[
V_{SWR} = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \quad V^+ = \frac{V_L + I_L Z_0}{2} \quad V^- = \frac{V_L - I_L Z_0}{2}
\]

Useful Equivalent Circuits for T-lines

Completely Discharged Line

Charged to a DC Steady State

General Equivalent Circuit

\[
\mu = \mu_r \mu_0 \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \quad \epsilon = \epsilon_r \epsilon_0 \quad \epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}
\]

\[
\epsilon_{\text{eff}} = 8.854 \times 10^{-12} \left[ \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \sqrt{1 + 12b/a} \right]
\]

\[
Z_0 = \frac{1}{2\pi} \mu / \epsilon_{\text{eff}} \ln \left( \frac{8b + a}{a + 4b} \right), \quad a < b
\]

\[
Z_0 = \sqrt{\mu / \epsilon_{\text{eff}}} \left[ \frac{1}{a/b + 1.393 + 0.667 \ln(a/b + 1.444)} \right], \quad a > b
\]