Name: ________________________

SSN: ________________________

ECE 3025: Electromagnetics
TEST 2 (Fall 2003)

- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** calculator, **closed** friend, **open** mind test. You should only have writing instruments on your desk when you take this test. If I find anything on your desk (excluding the test itself, writing instruments, and life-or-death medication) I will turn you in for an honor code violation. I am serious.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 50 minutes to complete this examination. When I announce a “last call” for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and social security number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

(1) **Short Answer Section** (24 points)

(a) __________________

A $\frac{\lambda}{4}$ transmission line with characteristic impedance Z_0 and load Z_L will appear to be an equivalent load of $Z_{in} = \boxed{\textit{Answer}}$ to a sinusoidal voltage source.

(b) __________________

The attenuation constant of a lossy transmission line has units of Nepers/m. However, most cabling specifications report loss in units of $\boxed{\textit{Answer 2}}$.

(c) __________________

Write the condition that must hold for unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ if we are using a right-handed coordinate system.

(d) __________________

You measure parameters on a transmission line and (to your astonishment) transit time is 1s, radian frequency is 1 rad/s, and the wavelength is 1m. What is the total length of the line in meters?

(e) __________________

When we use phasors to characterize voltage and current on a transmission line, we are assuming that the line is in the $\boxed{\textit{Answer}}$ state.

(f) __________________

True or False: The phasor transform does not preserve linearity of signals.

(g) __________________

Evaluate the following: $(\vec{a} - \vec{b}) \cdot (\vec{a} \times \vec{b})$.

(h) __________________

When the cross product of two vectors evaluates to the zero vector, we know that those two vectors are $\boxed{\textit{Answer}}$.

(i) __________________ (1) __________________ (2)

A voltage, $v(t)$, is described by the function $7\cos(2\pi ft - \frac{\pi}{4})$. The phasor value of $v(t)$ is $\boxed{\textit{Answer 1}}$ and the envelope of $v(t)$ is $\boxed{\textit{Answer 2}}$.

(j) __________________ (1) __________________ (2)

Loss-per-meter on a transmission line tends to increase when the frequency of the excitation $\boxed{\textit{Answer 1}}$ due to the $\boxed{\textit{Answer 2}}$ effect.

(2) **Descriptive Answer Section** (20 points)
Write a **concise** answer to each question in the spaces provided beneath each problem statement. **Note:** Correct answers that are extremely verbose will be penalized.

(a) **Waves on a Transmission Line:** *A true story.* Some Motorola engineers were taking power amplifier measurements on the roof of a building, hooking a UHF amplifier to an antenna with some heavy-duty coaxial cable. The load of the antenna was matched to the coaxial line so that, under normal operation, there are no reflections. Several thousand Watts were sent down the line to the antenna for each test ($\beta \approx \frac{2\pi}{1 \text{ meter}}$ for this example). During one test, the engineers accidentally kinked part of the coaxial cable. For this test, no power was radiated by the antenna, the power amplifier blew up, and there were actually periodic places on the cable (before the kink) that bulged where high voltages had melted and burned the dielectric of the expensive coaxial cable. Explain, electromagnetically, what happened and how far apart the bulges were in the ruined cable. (10 points)

(b) **Spherical Coordinates:** We know that for spherical coordinates, the unit vectors that describe field components point in different directions in space, depending on the observation point. If you know that $\theta = 0$ and $\phi = 0$ for the observation point, what are $\hat{a}_r$, $\hat{a}_\phi$, and $\hat{a}_\theta$ in terms of Cartesian unit vectors? (10 points)

(3) **Transmission Line with Sinusoidal Excitation:** (30 points) Below are phasor-domain voltage and current to a transmission line operating with steady-state sinusoids with frequency f:

$$\begin{aligned} \tilde{v}(z) &= 100\exp(-j\beta[z-D]) + 50\exp(j\pi + j\beta[z-D]) \\ \tilde{i}(z) &= \exp(-j\beta[z-D]) - \frac{1}{2}\exp(j\pi + j\beta[z-D]) \end{aligned}$$

where $z = 0$ is the source side and $z = D$ is the load side. Perform the following analysis:

(a) In the equations above, circle the portion of the solution representing the backward-propagating current waveform. (5 points)

(b) In the equations above, box the forward-propagating *amplitude* of the voltage waveform. (5 points)

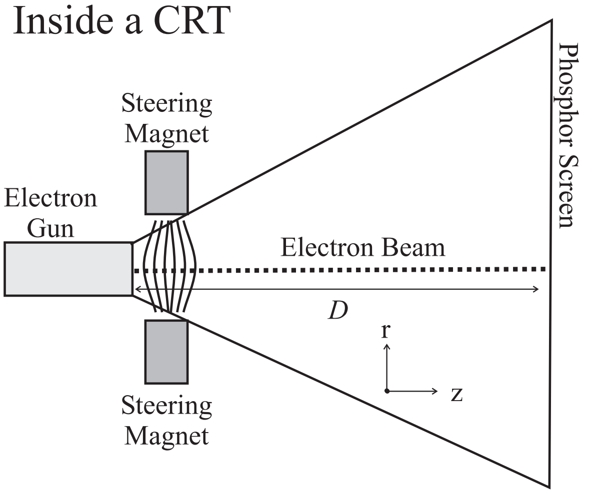
(c) Write a simplified expression for time-varying voltage evaluated at the front of the line ($z = 0$). Your answer should be in the time domain, $v(t)$. (5 points)

(d) What is the characteristic impedance, Z_0, of this line? (5 points)

(e) What is the VSWR of voltages on this line? (5 points)

(f) What is the load impedance, Z_L, for this line? (5 points)

(4) **Electron Gun in a TV:** (26 points) Below is a schematic of a cathode ray tube (CRT). To create images, an electron beam is swept across a phosphor screen using electromagnets. This electron beam can be approximated to be a thin, uniform line of free space charge with densisty, ρ_L. Answer the questions below based on this model.



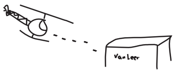
(a) If the beam starts at the cylindrical coordinate point (0,0,0) and ends at the point (0,0,D), what is the electric field as a function of space, $\vec{r}$, inside the CRT? (Set up the integral and simplify, but you do not have to evaluate the expression.) (10 points)

(b) Since the charges in the electron beam are not confined, there is a tendency of the beam to widen as it travels towards the phosphor screen. If a single electron (-1.60×10^{-19} C) is brought right very close to this beam (a small distance ρ), how much force is exerted upon it and in which direction? For close-in fields, you can approximate the line charge as infinite. (10 points)

(c) The electron gun takes static electrons and fires them in the $\hat{z}$ direction using strong electric fields. On the box in the diagram labeled "Electron Gun", place an arrow in the direction of the electric fields inside the box. (6 points)

Emag Rulz!!

Cheat Sheet



$$\frac{\partial^2 v(z,t)}{\partial z^2} = LC\frac{\partial^2 v(z,t)}{\partial t^2} \qquad \frac{\partial^2 i(z,t)}{\partial z^2} = LC\frac{\partial^2 i(z,t)}{\partial t^2} \qquad Z_0 = \sqrt{\frac{L}{C}} \qquad v_p = \frac{1}{\sqrt{LC}}$$

$$\lambda f = v_p \qquad \omega = 2\pi f \qquad \beta = \frac{2\pi}{\lambda} \qquad D = Tv_p$$

$$\text{Reflection: } \Gamma_{L,G} = \frac{Z_{L,G} - Z_0}{Z_{L,G} + Z_0} \qquad \text{Transmission: } 1 + \Gamma_{L,G}$$

$$\text{Phasor Transform: } A\cos(2\pi ft + \phi) \longrightarrow A\exp(j\phi)$$

$$\text{Reverse Transform: } \tilde{x} \longrightarrow \text{Real}\left\{\tilde{x}\exp(j2\pi ft)\right\}$$

$$\begin{aligned} v(z,t) &= V^+ f\left(t - \frac{z}{v_p}\right) + V^- g\left(t + \frac{z}{v_p}\right) \\ i(z,t) &= \frac{V^+}{Z_0} f\left(t - \frac{z}{v_p}\right) - \frac{V^-}{Z_0} g\left(t + \frac{z}{v_p}\right) \end{aligned}$$

$$\begin{aligned} v(z) &= V^+\exp(-\gamma z) + V^-\exp(+\gamma z) \\ i(z) &= \frac{V^+}{Z_0}\exp(-\gamma z) - \frac{V^-}{Z_0}\exp(+\gamma z) \end{aligned}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_{in} = Z_0\frac{Z_L + jZ_0\tan\beta D}{Z_0 + jZ_L\tan\beta D}$$

$$VSWR = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta_{AB} \qquad \vec{a}\times\vec{b} = \hat{a}_N|\vec{a}||\vec{b}|\sin\theta_{AB}$$

$$\text{Point Charge at the Origin: } \vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon r^2}\hat{a}_r$$

$$\vec{F} = Q\vec{E}$$

$$\begin{array}{c}\text{Charge}\\ \text{Distributions:}\end{array} \vec{E}(\vec{r}) = \int\limits_L \frac{\rho_L(\vec{r}')(\vec{r} - \vec{r}')dL}{4\pi\epsilon|\vec{r} - \vec{r}'|^3} = \iint\limits_S \frac{\rho_S(\vec{r}')(\vec{r} - \vec{r}')dS}{4\pi\epsilon|\vec{r} - \vec{r}'|^3} = \iiint\limits_V \frac{\rho_V(\vec{r}')(\vec{r} - \vec{r}')dV}{4\pi\epsilon|\vec{r} - \vec{r}'|^3}$$

$$\text{Infinite Line charge on z-axis: } \vec{E} = \frac{\rho_L}{2\pi\epsilon\rho}\hat{a}_\rho$$

$$\text{Infinite Sheet of charge on xy-plane: } \vec{E} = \frac{\rho_S}{2\epsilon}\hat{a}_z$$