Name: ______________________

GTID: ______________________

ECE 3025: Electromagnetics

TEST 2 (Fall 2006)

- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** calculator, **closed** friend, **open** mind test. You should only have writing instruments on your desk when you take this test. If I find anything on your desk (excluding the test itself, writing instruments, and life-or-death medication) I will turn you in for an honor code violation. I am serious.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 80 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

(1) **Short Answer Section** (**10 points**)

(a) ____________________

The *projection* of a vector into a specific direction is calculated by taking the *Answer* of the vector and a unit vector that points in the desired direction.

(b) ________________ (1) ________________ (2)

What are two types of *physical* loss mechanisms (contributors to α) in a transmission line?

(c) ________________ (1) ________________ (2)

In addition to per-unit-length series inductance, L, and shunt capacitance, C, what two circuit elements are added to our model for lossy transmission lines? Write words, not symbols.

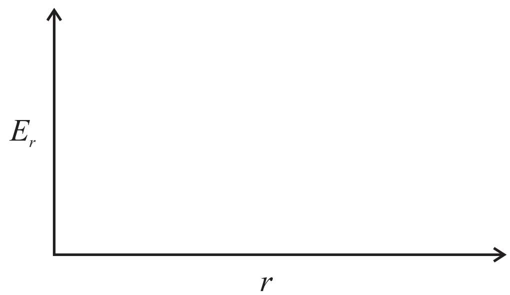
(2) **Vector Math:** A triangle is described by 3 vectors – $\vec{A}$, $\vec{B}$, and $\vec{C}$ – which define the vertices of the shape. In the space below, write an expression for a unit vector $\hat{n}$, in terms of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$, that is perpendicular to the plane of the triangle. (**16 points**)

(3) **Charge Distributions:** All of the field distributions in this problem are free-space and may be written in the following form:

$$\vec{E}(r,\phi,\theta) = E_r(r)\hat{r}$$

Make a rough sketch in the graph provided of $E_r(r)$ for the following charge distributions. (**24 points**)

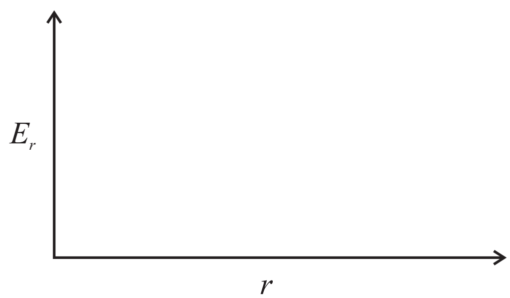
(a) At the origin is a +1 C point charge. Surrounding this charge is a spherical shell of uniform charge density centered at the origin at a radius of R and a total charge of -1 C. (**8 points**)



(b) There is a charge distribution in space of the form:

$$\rho_v(r,\phi,\theta) = \frac{\rho_0}{r^2}$$

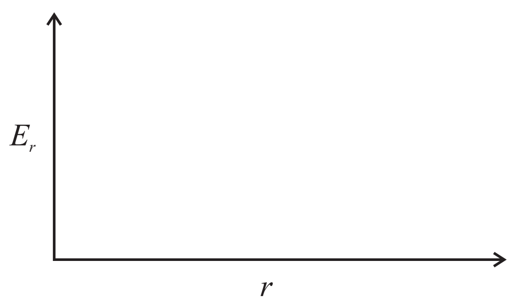
(**8 points**)



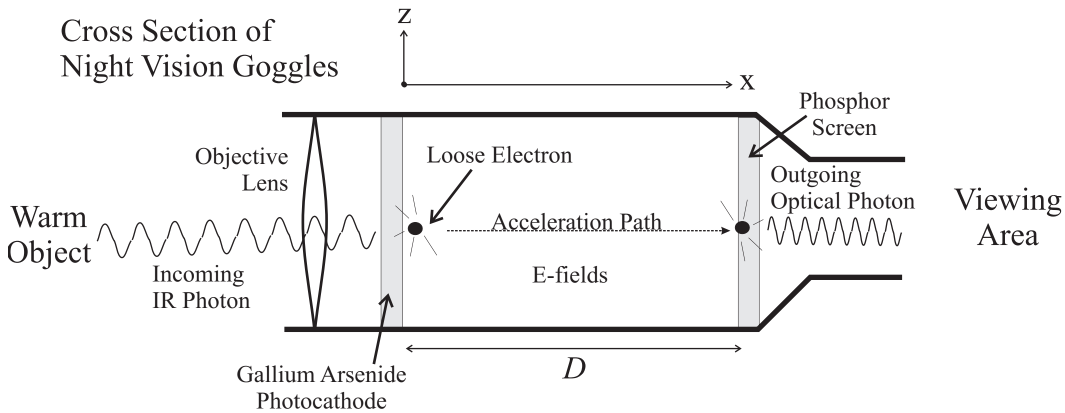
(c) There is a voltage distribution in space of the form:

$$V(r,\phi,\theta) = -r^2$$

(**8 points**)



(4) **Night Vision Goggles:** Commercial night vision goggles operate by receiving invisible infrared photons which, upon striking a thin Gallium-Arsenide *photocathode*, release a single electron into a vacuum acceleration chamber. In this chamber, strong electric fields accelerate the electron and smash it into a phosphor screen, where optical photons are released for the viewing person to see. (**20 points**)



The distance between the GaAs wafer and the phosphor screen is D. The E-field of the acceleration chamber is uniform with respect to space:

$$\vec{E}(\vec{r}) = E_0\hat{x}$$

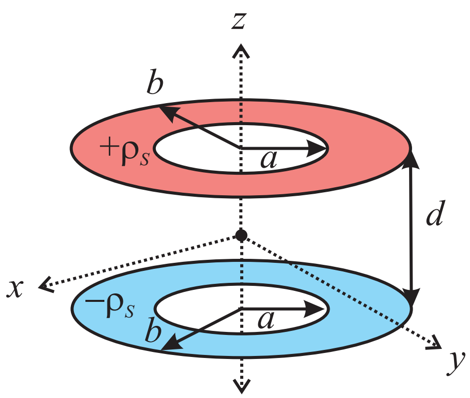
where E_0 is a constant (positive or negative). Answer the following questions (symbolically rather than numerically):

(a) Draw an arrow on the diagram above that points in the direction the electric field must point inside the acceleration chamber. (**5 points**)

(b) If the electron starts on the photocathode at rest, what is the minimum voltage drop across the chamber if the electron must strike the phosphor screen with at least W_0 Joules of kinetic energy (i.e. the field must do this much work on the electron in the acceleration chamber)? An electron has a charge of $-q$. (**10 points**)

(c) Based on your answer in part (b), what is the minimum value for electric field strength, E_0 (V/m), in the chamber? (**5 points**)

(5) **Charge Disks:** A pair of flat rings have uniform surface charge density with opposite polarities; a density of $+\rho_S$ is distributed on the top ring while a density of $-\rho_S$ is distributed on the bottom ring. The rings are separated by a distance d and have inner and outer radii of a and b, respectively. Given that the whole assembly is centered on the origin, find an expression for electrostatic field along the z-axis – i.e. find $\vec{E}(0, 0, z)$. *Simplify as much as possible* without evaluating the final integral(s). (**30 points**)



Bonus +5 points: Completely evaluate your answer, continuing on the back if necessary. Credit for this is all-or-nothing.

Formula Sheet

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta D}{Z_0 + jZ_L \tan \beta D}$$

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$\vec{F} = Q\vec{E} \qquad \text{Point Charge at the Origin: } \vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon r^2}\hat{r}$$

Charge Distributions: $$\vec{E}(\vec{r}) = \int_L \frac{\rho_L(\vec{r}')(\vec{r} - \vec{r}')dL}{4\pi\epsilon||\vec{r} - \vec{r}'||^3} = \iint_S \frac{\rho_S(\vec{r}')(\vec{r} - \vec{r}')dS}{4\pi\epsilon||\vec{r} - \vec{r}'||^3} = \iiint_V \frac{\rho_V(\vec{r}')(\vec{r} - \vec{r}')dV}{4\pi\epsilon||\vec{r} - \vec{r}'||^3}$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon r} = \int_L \frac{\rho_L(\vec{r}')dL}{4\pi\epsilon||\vec{r} - \vec{r}'||} = \iint_S \frac{\rho_S(\vec{r}')dS}{4\pi\epsilon||\vec{r} - \vec{r}'||} = \iiint_V \frac{\rho_V(\vec{r}')dV}{4\pi\epsilon||\vec{r} - \vec{r}'||}$$

Variable of integration: $\vec{r}'$ Point of observation: $\vec{r}$

$$\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z} = \frac{\partial V}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z} = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r \sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \qquad \nabla \times \vec{E} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{D} = \epsilon\vec{E} \qquad \vec{E} = -\nabla V \qquad \nabla \cdot \vec{D} = \rho_v \qquad V_B - V_A = -\int_A^B \vec{E} \cdot d\hat{l}$$

$$\text{Cross Product: } \vec{a} \times \vec{b} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \qquad \text{Dot Product: } \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\text{Sphere Volume: } \frac{4\pi}{3}r^3 \qquad \text{Sphere Surface Area: } 4\pi r^2$$