Practice Questions for Test 3

ECE 3025: Electromagnetics

Note: Below is a list of all reject test questions I compiled while writing the original test for this class. This document is only meant for practice. The questions on the in-class test ARE NOT as numerous as those contained by this document.

(1) **Short Answer Section**

(a) ________________

The curl of the static electric field is $\boxed{\textit{Answer}}$.

(b) ________________

The divergence of the magnetic field is $\boxed{\textit{Answer}}$.

(c) ________________

According to $\boxed{\textit{Answer}}$ theorem, the curl of a vector field across an area is equal to the line integration of that field around the perimeter.

(d) ________________

$\boxed{\textit{Answer}}$ law states that voltage is proportional to the time-change in magnetic flux.

(e) ________________

$\boxed{\textit{Answer}}$ law states that the closed line integral of magnetic field is equal to the enclosed current.

(f) __________ (1) __________ (2) __________ (3) __________ (4)

The charge quantities Q (point charge), ρ_L (line charge density), ρ_S (surface charge density), and ρ_V (volume charge density), have units of $\boxed{\textit{Answer 1}}$, $\boxed{\textit{Answer 2}}$, $\boxed{\textit{Answer 3}}$, and $\boxed{\textit{Answer 4}}$, respectively.

(g) __________ (1) __________ (2) __________ (3)

The current quantities $Id\vec{L}$ (line current charge), $\vec{K}$ (surface current density), and $\vec{J}$ (volume current density), have units of $\boxed{\textit{Answer 1}}$, $\boxed{\textit{Answer 2}}$, and $\boxed{\textit{Answer 3}}$, respectively.

(h) ________________

The point (differential) form of Ampere's law is $\boxed{\textit{Answer}}$.

(i) ____________________

True or False: curl and divergence vector operations are used solely in the field of electromagnetics.

(j) ____________________

The divergence of the gradient of a vector is also called the $\boxed{\textit{Answer}}$ operator.

(k) ____________________ (1) ____________________ (2)

The Laplacian operator operates on a $\boxed{\textit{Answer 1: vector/scalar}}$ quantity and results in a $\boxed{\textit{Answer 2: vector/scalar}}$ quantity.

(l) ____________________ (1) ____________________ (2)

The curl operator operates on a $\boxed{\textit{Answer 1: vector/scalar}}$ quantity and results in a $\boxed{\textit{Answer 2: vector/scalar}}$ quantity.

(m) ____________________ (1) ____________________ (2)

The divergence operator operates on a $\boxed{\textit{Answer 1: vector/scalar}}$ quantity and results in a $\boxed{\textit{Answer 2: vector/scalar}}$ quantity.

(n) ____________________ (1) ____________________ (2)

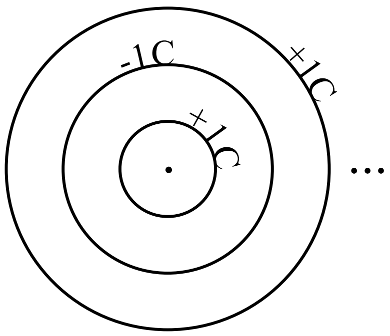
The gradient operator operates on a $\boxed{\textit{Answer 1: vector/scalar}}$ quantity and results in a $\boxed{\textit{Answer 2: vector/scalar}}$ quantity.

(o) ____________________

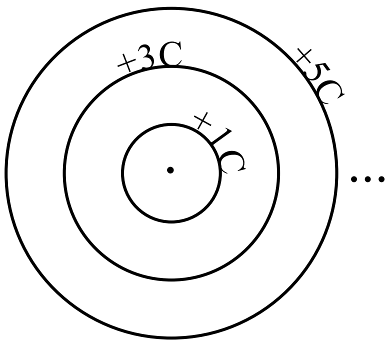
$\boxed{\textit{Answer}}$'s law states that the surface integral of electric flux density is equal to the total enclosed charge.

(2) **Descriptive Answer Section**

(a) **E-Fields of Charge Shells:** Imagine a scenario of successive spherical shells of uniform charge density centered at the origin, increasing in size, and alternating +1 C and -1 C in total charge. Thus, the first shell has radius R and contains a total charge of +1 C spread out uniformly on its surface. The second shell has radius $2R$ and contains a total charge of -1 C. The third shell has radius $3R$ and total charge +1 C. This pattern continues to infinity. Sketch the magnitude of the E-field as a function of distance from the origin, r. You do not have to show exact amplitudes – only the basic behavior with respect to r.



(b) **Gauss's Law:** Imagine a scenario of successive spherical shells of uniform charge density centered at the origin, increasing in size by 1m and total charge by +2 C on every shell. Thus, the first shell has radius 1m and contains a total charge of +1 C spread out uniformly on its surface. The second shell has radius 2m and contains a total charge of +3 C. The third shell has radius 3m and total charge +5 C. This pattern continues to infinity. What is the value of the electric field very far from the origin (large r).



(c) **Field Properties:** Below are a list of "mystery fields". Determine which could be a valid candidate for either a static electric field or magnetic field. Mark **E** for electric field only, **H** for magnetic field only, **B** for both electric and magnetic field, and **N** for neither. Write a *brief* explanation for each justifying your answer.

(i) $6\hat{a}_x + 7\hat{a}_y + 8\hat{a}_z$

(ii) $xyz\hat{a}_x$

(iii) $\frac{1}{x}\hat{a}_x - \frac{1}{y}\hat{a}_y$

(iv) $\frac{1}{y}\hat{a}_x - \frac{1}{x}\hat{a}_y$

(d) **Operators in Other Coordinate Systems:**
Explain why you cannot take the divergence of a vector in spherical coordinates by straight-summing the partial derivatives of each component:

$$\textbf{Wrong: } \nabla \cdot \vec{D} = \frac{\partial D_\rho}{\partial \rho} + \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_\theta}{\partial \theta}$$

Work-out Problem Section

(3) **Capacitance of a MOSFET:** An n-channel MOSFET is shown in the diagram below. It consists of metal gate, drain, and source contacts, an insulating layer of SiO_2, and an n- and p-type Si substrate. The width of the SiO_2 layer is d_1 and the cumulative width of the two Si substrates is d_2. The SiO_2 layer has permittivity ϵ_1 and the substrates both have permittivity ϵ_2. The gate has surface area (in the xy-plane) of A and is held at a potential V above ground, which is at the bottom of the substrates. Prove that the gate capacitance (with respect to the ground plane beneath the stubstrates) is given by the following expression:

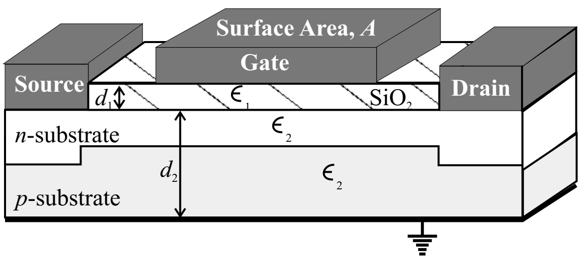
$$C = \frac{A\epsilon_1\epsilon_2}{\epsilon_2 d_1 + \epsilon_1 d_2}$$

Hint: The definition of capacitance is $C = Q/V$. You may approximate this scenario as a parallel-plate capacitor where the electric flux density underneath the gate is given by

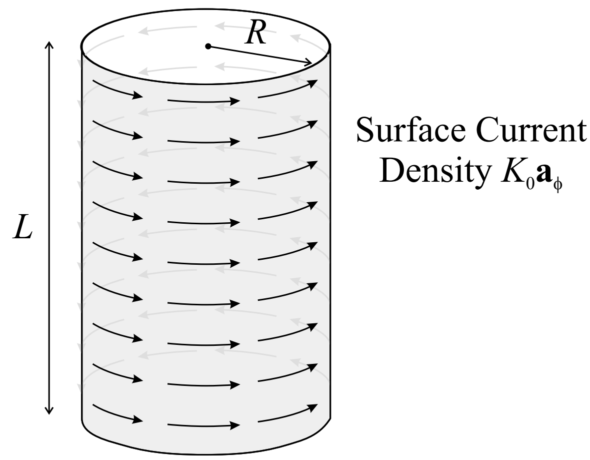
$$\vec{D} = -\rho_S \hat{a}_z$$

where ρ_S is the surface charge density on the gate.

n-Channel Depletion MOSFET



(4) **Cylinder Current:** A sheet of constant surface current density, J_0, is circulating in the $+\hat{a}_\phi$ direction around the surface of a cylinder with radius R. The cylinder is centered on the origin with total length L. Set up and simplify the integral for magnetic field at an arbitrary point in space (x,y,z) using the Biot-Savart relationship, but do not evaluate it.



(5) **Crazy Continuity:** Below is a circuit with four different regions of current density. The first is an $L \times W$ sheet of constant current density. The second is a length of solid constant current density carried in a square $W \times W$ cross-section. The third is a length of solid constant current density carried in a cylindrical tube of radius $\frac{W}{2}$. Finally, a line current of I completes the circuit. If all of these currents are to solve the continuity equation, what are the values of K_0, J_1, and J_2 in terms of I and geometrical dimensions? You may assume that a sheet of current exists in the yz plane between each transition regional that also satisfies the continuity equation.