Name: ______________________

GTID: ______________________

ECE 3025: Electromagnetics

TEST 3 (Fall 2003)

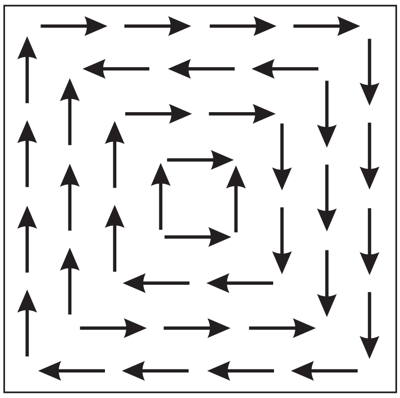
- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** calculator, **closed** friend, **open** mind test. You should only have writing instruments on your desk when you take this test. If I find anything on your desk (excluding the test itself, writing instruments, and life-or-death medication) I will turn you in for an honor code violation. I am serious.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 50 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and social security number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ______________________________________

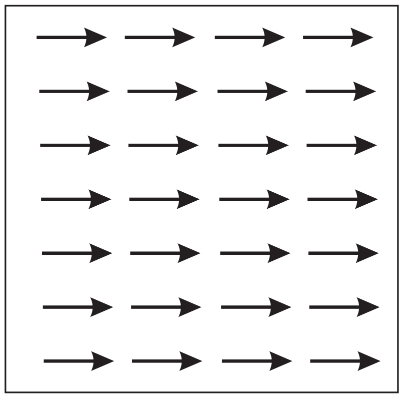
I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

(1) **Field Properties:** Below are 4 sketches of vector fields. One could be a **M**agnetic field, one could be an **E**lectric field, one could be **B**oth, and one could be **N**either. Below each sketch, label which type of field it is (with the letter M, E, B, or N) and a *brief* explanation of why in terms of basic vector and field properties (divergence, curl, gradient, charge, current, etc.) Assume that these 2D sketches represent cross-sections of fields that are constant with respect to the third dimension. (20 points)

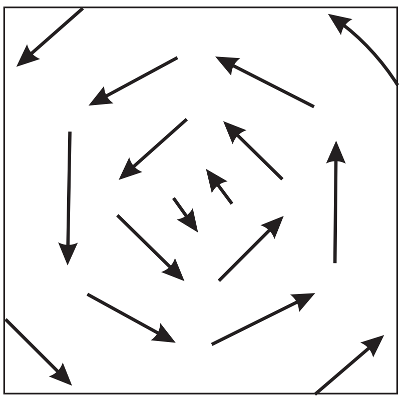
Irregular Square Field



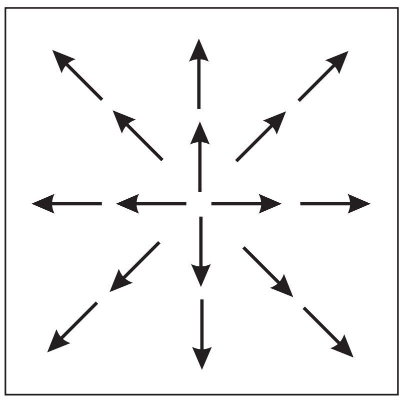
Constant Field



Round Field



Emanating Field

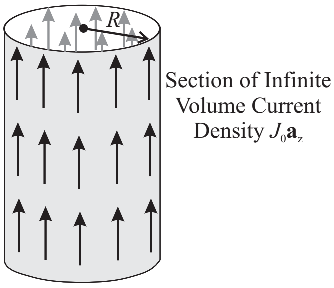


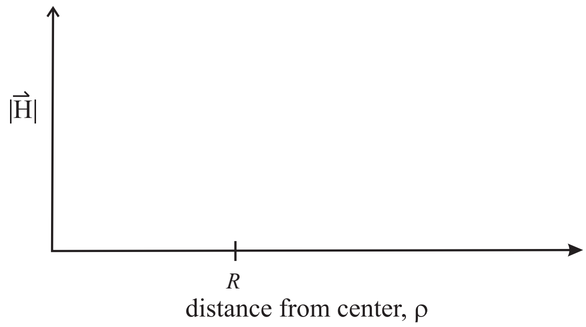
(2) **Thick Wire Current:** Rather than approximate a current in a wire as a line current, it is more realistic to treat it as a volume charge over a cylindrical area with the following current density:

$$\vec{J} = \begin{cases} J_0 \hat{a}_z & \text{for } \rho \leq R \\ 0 & \text{for } \rho > R \end{cases}$$

Answer the following questions based on this scenario.

(a) Use Ampere's law to derive a solution for $\vec{H}$ at all points in space and sketch the magnitude of your answer as a function of ρ in the space below (15 points):





(b) If the conductivity of the wire is σ, write an expression for $\vec{E}$ inside the wire. (5 points)

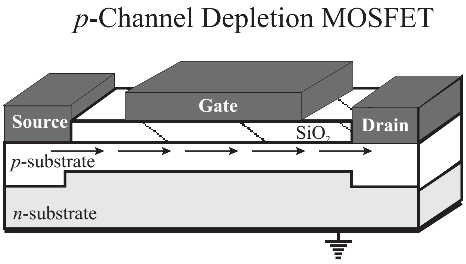
(c) Prove that your answer in part (a) satisfies Maxwell's curl equation. (10 points)

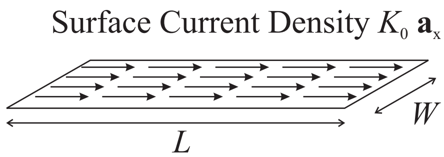
Hint: the curl of a vector in cylindrical coordinates is

$$\nabla \times \vec{H} = \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \hat{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) \hat{a}_\phi + \frac{1}{\rho} \left(\frac{\partial (\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} \right) \hat{a}_z$$

(3) **MOSFET Current:** The current between the source and drain of a depletion MOSFET can be crudely modeled as a rectangular sheet current as shown below.

(a) Set up and simplify the integral to solve for $\vec{H}(x, y, z)$ using the Biot-Savart relationship, *but do not evaluate it.* Set up the integral as if the current sheet were centered on the origin, lying in the xy-plane. (15 points)





(b) Why is it incomplete to model magnetic field solely due to this current sheet? (5 points)

(4) **TV Buttons:** You purchase a new CRT computer monitor and notice 6 buttons on the bottom display that modify the display in certain ways. They are labeled *contrast*, *sharpness*, *horz. shift*, *vert. shift*, *width*, and *height*. Each function knob is tied to a variable component in the CRT circuit schematic (labeled with bold letters **A-F**) contained on the following page. In the space below, *briefly* write which screen function is tied to which variable circuit component and explain your answer using basic electromagnetic principles. Verbose answers will be penalized. (30 points)

A:

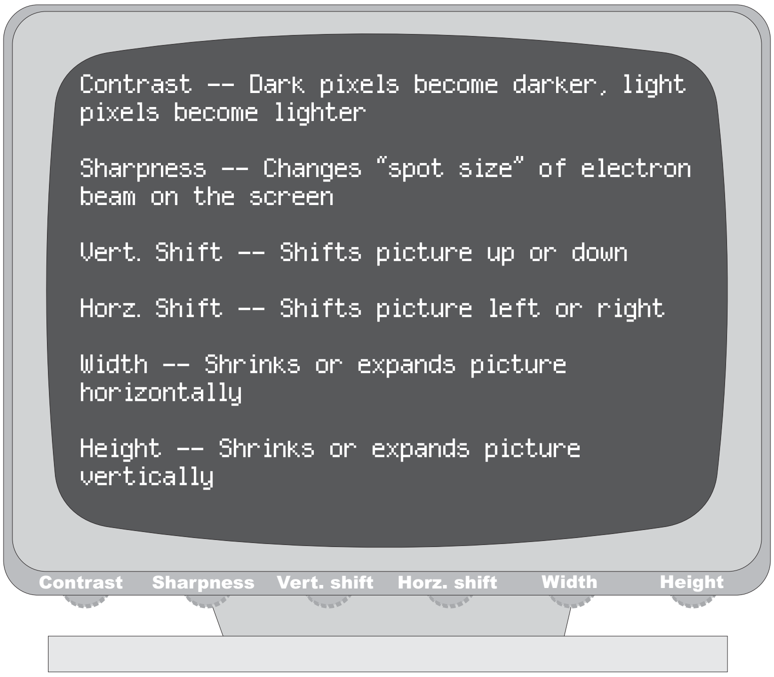
B:

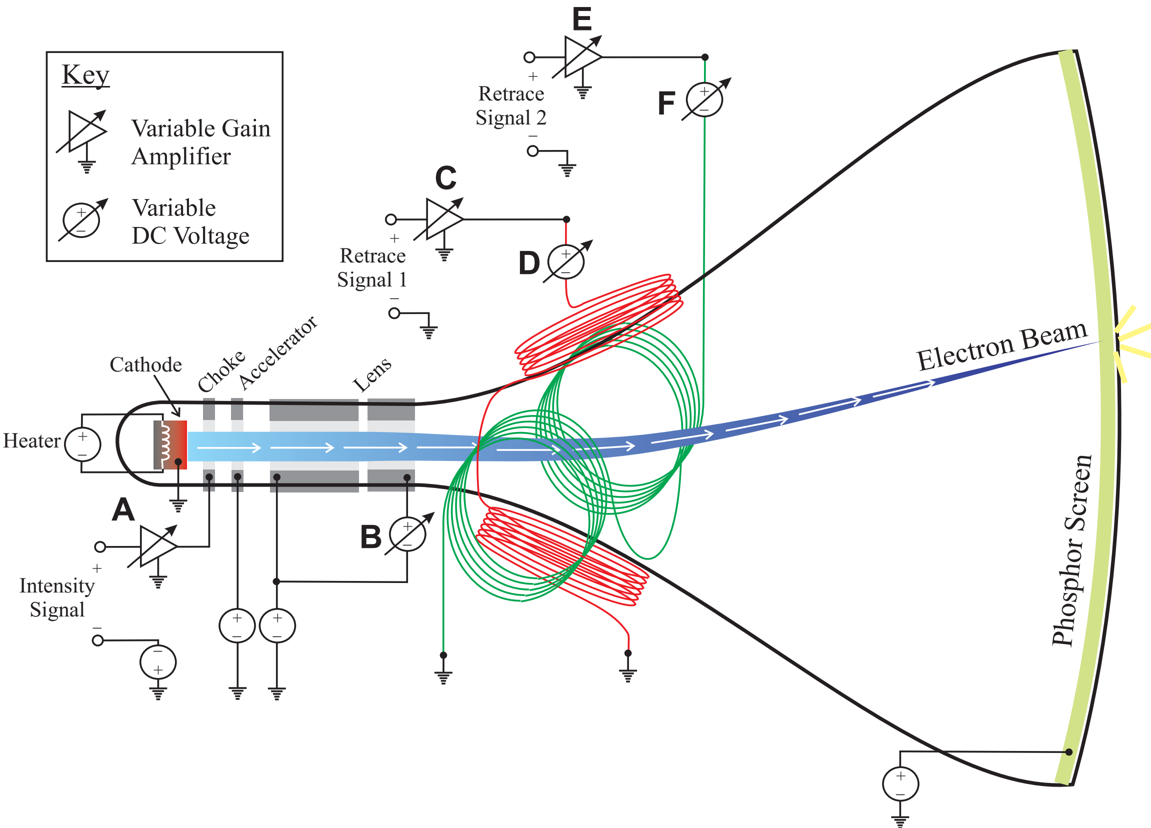
C:

D:

E:

F:


Contrast -- Dark pixels become darker, light pixels become lighter
Sharpness -- Changes "spot size" of electron beam on the screen
Vert. Shift -- Shifts picture up or down
Horz. Shift -- Shifts picture left or right
Width -- Shrinks or expands picture horizontally
Height -- Shrinks or expands picture vertically
Contrast
Sharpness
Vert. shift
Horz. shift
Width
Height

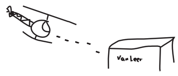

Key
Variable Gain Amplifier
Variable DC Voltage
E
F
Retrace Signal 2
C
D
Retrace Signal 1
Cathode
Choke
Accelerator
Lens
Heater
A
B
Intensity Signal
Electron Beam
Phosphor Screen

(5)

Emag Rulz!!

Cheat Sheet

(∇×)y hair 



$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta_{AB} \qquad \vec{a}\times\vec{b} = \hat{a}_N|\vec{a}||\vec{b}|\sin\theta_{AB} = \det\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon r^2}\hat{a}_r = \int_L \frac{\rho_L(\vec{r}')(\vec{r}-\vec{r}')dL}{4\pi\epsilon|\vec{r}-\vec{r}'|^3} = \iint_S \frac{\rho_S(\vec{r}')(\vec{r}-\vec{r}')dS}{4\pi\epsilon|\vec{r}-\vec{r}'|^3} = \iiint_V \frac{\rho_V(\vec{r}')(\vec{r}-\vec{r}')dV}{4\pi\epsilon|\vec{r}-\vec{r}'|^3}$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon r} = \int_L \frac{\rho_L(\vec{r}')dL}{4\pi\epsilon|\vec{r}-\vec{r}'|} = \iint_S \frac{\rho_S(\vec{r}')dS}{4\pi\epsilon|\vec{r}-\vec{r}'|} = \iiint_V \frac{\rho_V(\vec{r}')dV}{4\pi\epsilon|\vec{r}-\vec{r}'|}$$

$$\vec{H}(\vec{r}) = \int_L \frac{Id\vec{L}\times(\vec{r}-\vec{r}')}{4\pi|\vec{r}-\vec{r}'|^3} = \iint_S \frac{\vec{K}(\vec{r}')\times(\vec{r}-\vec{r}')dS}{4\pi|\vec{r}-\vec{r}'|^3} = \iiint_V \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')dV}{4\pi|\vec{r}-\vec{r}'|^3}$$

Variable of integration: $\vec{r}'$ Point of observation: $\vec{r}$

Infinite Line charge on z-axis: $\vec{E} = \dfrac{\rho_L}{2\pi\epsilon\rho}\hat{a}_\rho$

Infinite Sheet of charge on xy-plane: $\vec{E} = \dfrac{\rho_S}{2\epsilon}\hat{a}_z$

Infinite Line Current on z-axis: $\vec{H} = \dfrac{I}{2\pi\rho}\hat{a}_\phi$

$$\vec{E} = -\nabla V \qquad V = \int \vec{E}\cdot d\vec{L}$$

Maxwell's Equations (Static Point Form)

$$\begin{aligned} \nabla\times\vec{E} &= 0 & \qquad \nabla\times\vec{H} &= \vec{J} \\ \nabla\cdot\vec{D} &= \rho_V & \qquad \nabla\cdot\vec{B} &= 0 \end{aligned}$$

Maxwell's Equations (Static Integral Form)

$$\oint_L \vec{E}\cdot d\vec{L} = 0 \qquad \oint_L \vec{H}\cdot d\vec{L} = \underbrace{\iint_A \vec{J}\cdot d\hat{n}}_{I}$$

$$\oiint_A \vec{D}\cdot d\hat{n} = \iiint_V \rho_V\, dV \qquad \oiint_A \vec{B}\cdot d\hat{n} = 0$$

$$\nabla^2 V = \nabla\cdot\nabla V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \qquad \nabla\times\vec{E} = \det\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{D} = \epsilon\vec{E} \qquad \vec{B} = \mu\vec{H} \qquad \nabla\cdot\vec{J} = -\frac{\partial\rho_v}{\partial t} \qquad \vec{J} = \sigma\vec{E} \qquad R = \frac{L}{A\sigma}$$