Name: ______________________________

GTID: ______________________________

ECE 3025: Electromagnetics

TEST 3 (Fall 2005)

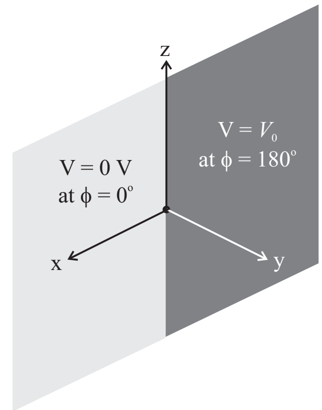
- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** calculator, **closed** friend, **open** mind test. You should only have writing instruments on your desk when you take this test. If I find anything on your desk (excluding the test itself, writing instruments, and life-or-death medication) I will turn you in for an honor code violation. I am serious.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 50 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

(1) **Laplace's Equation:** Two semi-infinite conductive plates are joined at the z-axis and held at 0 and V_0 Volts as shown in the diagram below. Answer the following questions based on this scenario.

(a) Use Laplace's equation to derive the result $V(\rho,\phi,z) = \frac{V_0|\phi|}{\pi}$ for the region $-\pi \le \phi \le \pi$. (15 points)

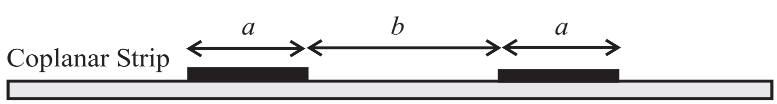


(b) Using the voltage in (a), show that the electric field distribution in space is the following: (10 points)

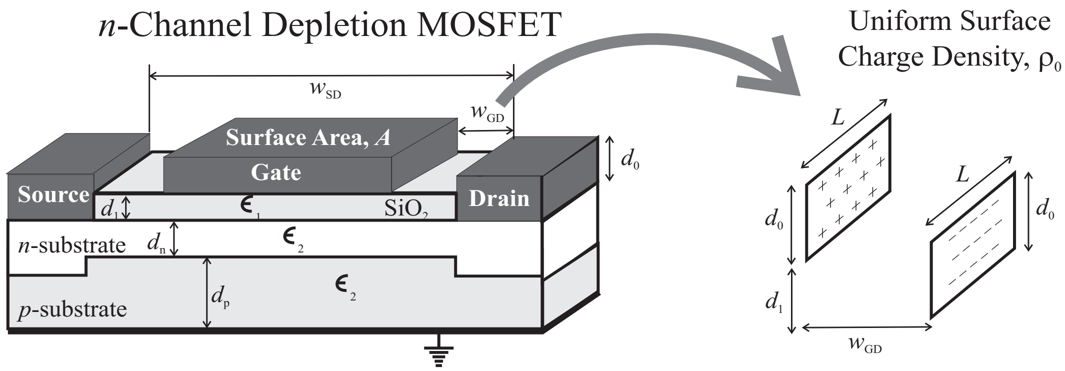
$$\vec{E}(\rho,\phi,z) = \begin{cases} -\frac{V_0}{\rho\pi}\hat{\phi} & 0 < \phi < \pi \\ \frac{V_0}{\rho\pi}\hat{\phi} & -\pi < \phi < 0 \end{cases} \quad \textbf{or} \quad \vec{E}(\rho,\phi,z) = \frac{2V_0}{\rho\pi}\left[\frac{1}{2} - \mathrm{u}(\phi)\right]\hat{\phi}$$

(c) What is the surface charge density on the plates? (10 points) Hint: $\rho_v(\vec{r}) = \rho_s(\rho,z)\delta(\phi)$

(d) **Bonus Challenge:** Use these results to approximate the per-unit-length capacitance of a coplanar strip transmission line with trace width a and separation distance b. (+5 points)



(2) **MOSFET Charge:** The charge between the gate and drain of a depletion MOSFET can be crudely modeled as two rectangular sheets of uniform surface charge density, $\pm\rho_0$, as shown below. As an electromagnetics engineer for RFMP (ridiculously fast micro-processors) Inc, you must estimate the voltage around this circuit as a key part of breakdown voltages and parasitic capacitances of the device.

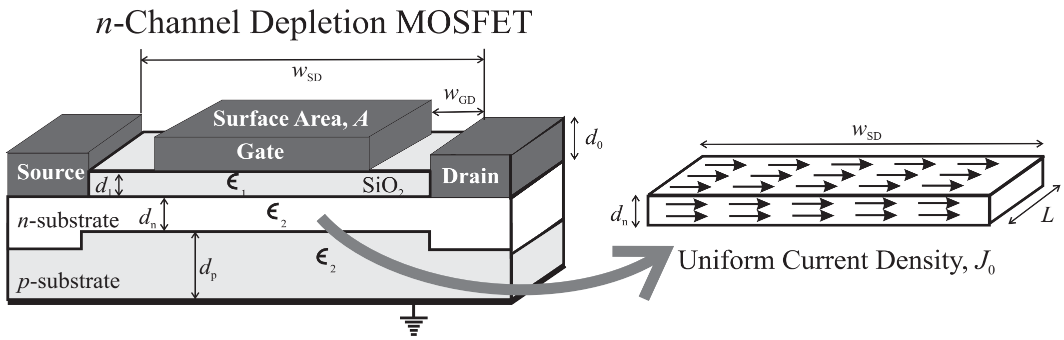


(a) In the space below, clearly sketch and label the geometry and orientation for the charge distributions that you have chosen to evaluate $V(x, y, z)$. (5 points)

(b) Based on your answer in (a), set up and simplify the integral to solve for $V(x, y, z)$, *but do not evaluate it*. Approximate the medium as free space with ϵ_0. (20 points)

(c) You enter the integral from (b) into the computer and solve for $V(\vec{r})$ in all 3D space. Write an expression for calculating $\vec{E}(\vec{r})$ in all of 3D space based on your result in (b). (5 points)

(3) **MOSFET Current:** The current between the source and drain of a depletion MOSFET can be crudely modeled as a rectangular prism of constant current density as shown below. As an electromagnetics engineer for RFMP (ridiculously fast micro-processors) Inc, you must estimate the magnetic fields around this circuit as a key part of determining parasitic inductances of the device.



(a) Write an expression for the total current, I, through the MOSFET. (5 points)

(b) In the space below, clearly sketch and label the geometry and orientation for the current density that you have chosen to evaluate $\vec{H}(x, y, z)$. (5 points)

(c) Based on your answer in (a), set up and simplify the integral to solve for $\vec{H}(x, y, z)$ using the Biot-Savart relationship, *but do not evaluate it.* (20 points)

(d) Why is it incomplete to model magnetic field solely due to this current sheet? (5 points)

Formula Sheet

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon r^2}\hat{r} = \int\limits_L \frac{\rho_L(\vec{r}')(\vec{r}-\vec{r}')dL}{4\pi\epsilon||\vec{r}-\vec{r}'||^3} = \iint\limits_S \frac{\rho_S(\vec{r}')(\vec{r}-\vec{r}')dS}{4\pi\epsilon||\vec{r}-\vec{r}'||^3} = \iiint\limits_V \frac{\rho_V(\vec{r}')(\vec{r}-\vec{r}')dV}{4\pi\epsilon||\vec{r}-\vec{r}'||^3}$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon r} = \int\limits_L \frac{\rho_L(\vec{r}')dL}{4\pi\epsilon||\vec{r}-\vec{r}'||} = \iint\limits_S \frac{\rho_S(\vec{r}')dS}{4\pi\epsilon||\vec{r}-\vec{r}'||} = \iiint\limits_V \frac{\rho_V(\vec{r}')dV}{4\pi\epsilon||\vec{r}-\vec{r}'||}$$

$$\vec{H}(\vec{r}) = \int\limits_L \frac{Id\vec{L}\times(\vec{r}-\vec{r}')}{4\pi||\vec{r}-\vec{r}'||^3} = \iint\limits_S \frac{\vec{K}(\vec{r}')\times(\vec{r}-\vec{r}')dS}{4\pi||\vec{r}-\vec{r}'||^3} = \iiint\limits_V \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')dV}{4\pi||\vec{r}-\vec{r}'||^3}$$

Variable of integration: $\vec{r}'$ Point of observation: $\vec{r}$

Infinite Line charge on z-axis: $\vec{E} = \frac{\rho_L}{2\pi\epsilon\rho}\hat{\rho}$

Infinite Sheet of charge on xy-plane: $\vec{E} = \frac{\rho_S}{2\epsilon}\hat{z}$

Infinite Line Current on z-axis: $\vec{H} = \frac{I}{2\pi\rho}\hat{\phi}$

$$\vec{E} = -\nabla V \qquad V = \int \vec{E}\cdot d\vec{L}$$

$$\nabla^2 V = \nabla\cdot\nabla V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \qquad \nabla\times\vec{E} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{D} = \epsilon\vec{E} \qquad \vec{B} = \mu\vec{H} \qquad \nabla\cdot\vec{D} = \rho_v \qquad \vec{J} = \sigma\vec{E} \qquad R = \frac{L}{A\sigma}$$

Capacitance: $C = \frac{Q}{V}$ Capacitance (F/m): $C = \frac{\rho_L}{V}$

$$\mathrm{u}(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad \frac{d|x|}{dx} = 2\mathrm{u}(x) - 1 \qquad \delta(x) = \frac{d\mathrm{u}(x)}{dx}$$