Name: ____________________

GTID: ____________________

ECE 3025: Electromagnetics

TEST 3 (Spring 2008)

- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** calculator, **closed** friend, **open** mind test. You should only have writing instruments on your desk when you take this test. If I find anything on your desk (excluding the test itself, writing instruments, and life-or-death medication) I will turn you in for an honor code violation. I am serious.
- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.
- You have 80 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

(1) **Short Answer Section** (20 points)

(a) __________ __________ __________ __________
What are the four properties of a simple medium?

(b) ____________
The type of magnetic material behavior that results from a Lenz-law deformation of paired electron orbitals. This type of magnetism, which persists at very high frequency, leads to very weak, negative susceptibility with no permanent magnetic dipole moment in the material.

(c) ____________
The type of magnetic material behavior that results from complete alignment of *all* unpaired electron spins. These magnetic materials have extraordinarily high μ_r below the Curie temperature and can possess a permanent magnetic dipole.

(d) ____________
The type of magnetic material behavior that results from partial alignment of unpaired electron spins in an impressed field. A modest magnetic behavior that persists at high frequencies, does not support a permanent magnetic dipole, and degrades gracefully at higher temperatures.

(e) ____________
The type of magnetic material behavior that results from the alternating crystalline *rows* of paramagnetic atoms with different paramagnetic dipole moments. The net total magnetic moment in the material results in a $\mu_r > 1$ and a permanent magnetic dipole moment. The most famous example of this type of material is *magnetite* (form of iron oxide).

(f) ____________
This type of magnetism results in $\mu_r > 1$, even at UHF and microwave frequencies. It is formed by placing ferromagnetic nano-particles in suspension.

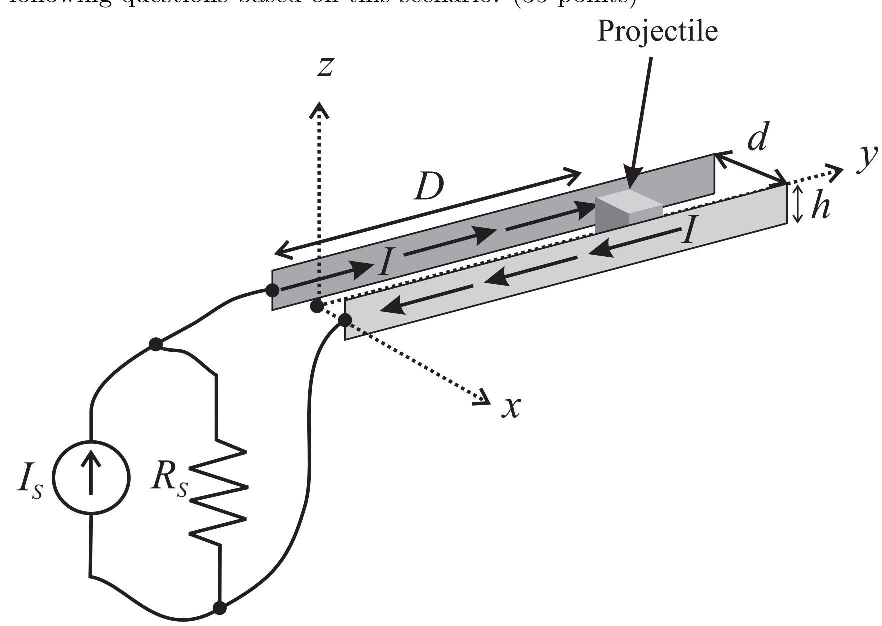
(g) ____________
The type of magnetic material behavior that results from alternating paramagnetic atoms in a crystal lattice, each with different magnetic dipole moments. The net total magnetic moment in the material results in a $\mu_r > 1$ and a permanent magnetic dipole moment.

(2) **Continuity Equation:** Often included as the "fifth Maxwell equation", the continuity equation,

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

can be derived from the original four differential-form equations. Perform this derivation (10 points) and explain what the relationship means in words (10 points).

(3) **Rail Gun Magnetic Fields:** A rail gun operates by sending current through two rails shown in the diagram below. A metal projectile completes the circuit and the resulting magnetic forces acting on the current within the projectile propel it down the rails. In the diagram below, the y-axis bisects the two rails, which are separated by a distance d. Answer the following questions based on this scenario. (35 points)



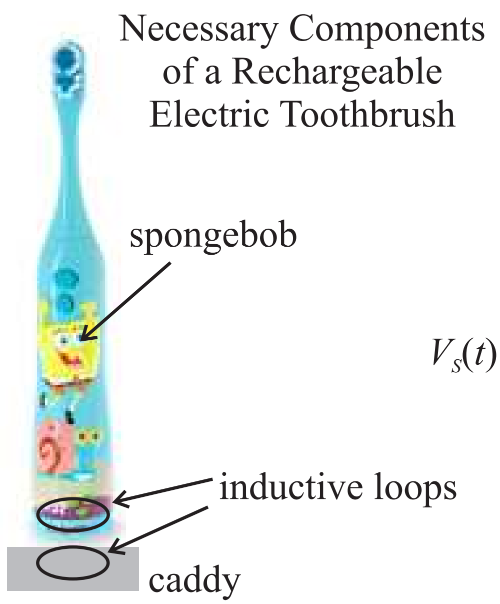
(a) If you model the total current I as an ideal line current at the center of each rail (on the xy-plane), estimate the magnetic flux density $\vec{B}(0, D, 0)$ operating on a projectile that has traveled a length D down the line. Neglect contributions from currents that are not on the rails. Simplify as much as possible, but you do not have to evaluate the final integral. (20 points)

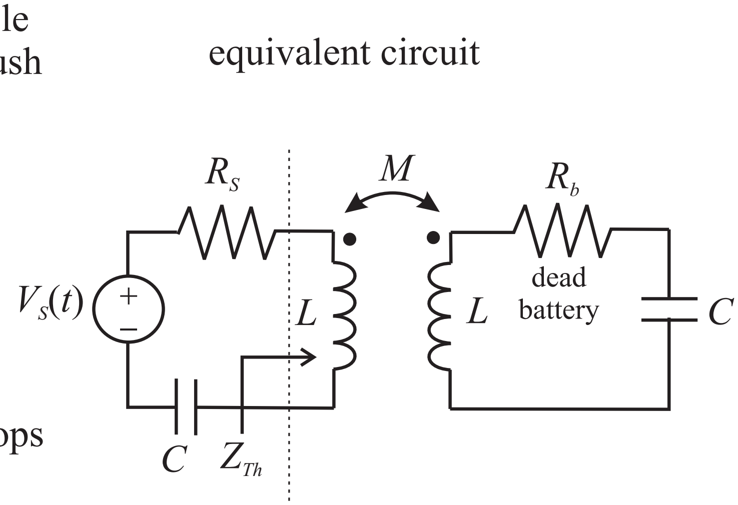
(b) Write an expression for the true uniform surface current density vector $\vec{K}$ (Amps/m) on the rails in terms of I (Amps). (5 points)

(c) If the current source in the diagram is ideal, explain what will happen to the rail current I as the projectile travels further down the line. (5 points)

(d) Ideally, an infinite R_s will allow the current source to couple maximum power into the rails for propulsion. However, what properties of the rail might make a finite resistance desirable? (Hint: think transmission lines) (5 points)

(4) **Inductive Toothbrush Charger:** An electric toothbrush uses inductive charging to recharge its batteries without requiring an electrical connection. The toothbrush has a single wire loop at its base that, when placed in the caddy, inductively couples to another identical current loop connected to a source. Answer the questions below based on this scenario. (25 points)





(a) If you are given a system operating frequency of f, write an expression for the value C that optimizes charging. (5 points)

(b) If $M \approx L$ in this system, what does this say – if anything – about the spatial distance between the two loops? (5 points)

(c) If you are given a system operating frequency of f, write an expression for the efficiency of the system. This is the ratio of the power flowing into the charging batteries R_b to the power flowing out of the charging source. Does efficiency increase or decrease as frequency increases? (15 points)

Formula Sheet

Vector Identity: $\nabla \cdot (\nabla \times \vec{A}) = 0$

$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$ Point Charge at the Origin: $\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon r^2}\hat{r}$

$$\vec{H}(\vec{r}) = \int\limits_L \frac{Id\vec{L} \times (\vec{r} - \vec{r}')}{4\pi||\vec{r} - \vec{r}'||^3} = \iint\limits_S \frac{\vec{K}(\vec{r}') \times (\vec{r} - \vec{r}')dS}{4\pi||\vec{r} - \vec{r}'||^3} = \iiint\limits_V \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')dV}{4\pi||\vec{r} - \vec{r}'||^3}$$

Variable of integration: $\vec{r}'$ Point of observation: $\vec{r}$

$$\vec{D} = \epsilon\vec{E} \qquad \vec{E} = -\nabla V \qquad \nabla \cdot \vec{D} = \rho_v \qquad \vec{B} = \mu\vec{H}$$

Cross Product: $\vec{a} \times \vec{b} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \|\vec{a}\|\|\vec{b}\| \sin\theta_{ab}\hat{n}$

Dot Product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = \|\vec{a}\|\|\vec{b}\| \cos\theta_{ab}$

Inductance: $L = \frac{\text{Magnetic Flux}}{\text{Current}}$ $M = \frac{\text{Shared Magnetic Flux}}{\text{Current}}$

$$Z_{Th} = j2\pi fL + \frac{4\pi^2 f^2 M^2}{R_b + j\left(2\pi fL - \frac{1}{2\pi fC}\right)}$$

$$\boxed{\begin{array}{ll} \nabla \times \vec{H}(\vec{r},t) = \frac{\partial \vec{D}(\vec{r},t)}{\partial t} + \vec{J}(\vec{r},t) & \nabla \cdot \vec{B}(\vec{r},t) = 0 \\ \nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t} & \nabla \cdot \vec{D}(\vec{r},t) = \rho_v(\vec{r},t) \end{array}}$$

DIVERGENCE

CARTESIAN $\nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

CYLINDRICAL $\nabla \cdot \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

SPHERICAL $\nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$

GRADIENT

CARTESIAN $\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$

CYLINDRICAL $\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$

SPHERICAL $\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$

CURL

CARTESIAN $\nabla \times \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z$

CYLINDRICAL $\nabla \times \mathbf{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right]\mathbf{a}_z$

SPHERICAL $\nabla \times \mathbf{H} = \frac{1}{r\sin\theta}\left[\frac{\partial(H_\phi \sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial(rH_\phi)}{\partial r}\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right]\mathbf{a}_\phi$

LAPLACIAN

CARTESIAN $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

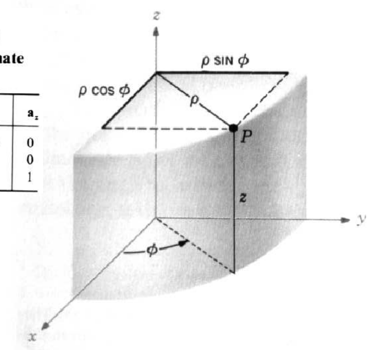
CYLINDRICAL $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

SPHERICAL $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$

Cylindrical Coordinates

Dot products of unit vectors in cylindrical and cartesian coordinate systems

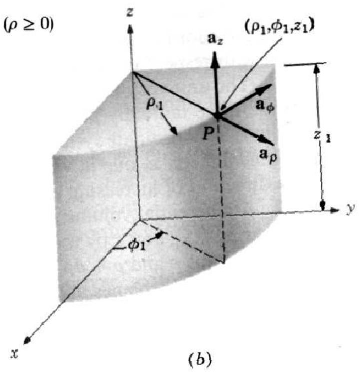
	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1



$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$

$\phi = \tan^{-1}\frac{y}{x}$

$z = z$

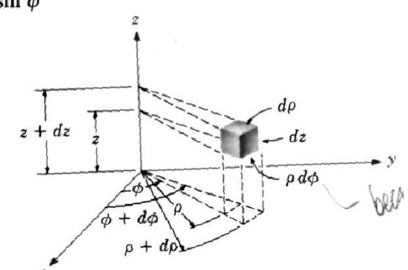


(b)

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

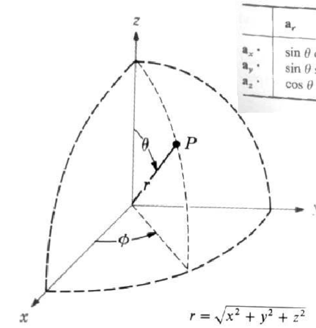


(c)

Spherical Coordinates

Dot products of unit vectors in spherical and cartesian coordinate systems

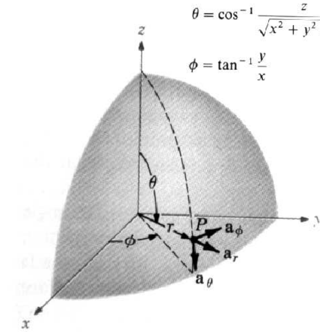
	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0



$r = \sqrt{x^2 + y^2 + z^2} \quad (r \geq 0)$

$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}} \quad (0^\circ \leq \theta \leq 180^\circ)$

$\phi = \tan^{-1}\frac{y}{x}$



$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$