Name: ______________________________

GTID: ______________________________

ECE 3025: Electromagnetics
TEST 3 (Fall 2009)

- Please read all instructions before continuing with the test.

- This is a **closed** notes, **closed** book, **closed** calculator, **closed** friend, **open** mind test. You should only have writing instruments on your desk when you take this test. If I find anything on your desk (excluding the test itself, writing instruments, and life-or-death medication) I will turn you in for an honor code violation. I am serious.

- Show all work. (It helps me give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.

- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.

- All necessary mathematical formulas are included either in the problem statements or the last few pages of this test.

- You have 80 minutes to complete this examination. When I announce a "last call" for examination papers, I will leave the room in 5 minutes. The fact that I do not have your examination in my possession will not stop me.

- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.

- Have a nice day!

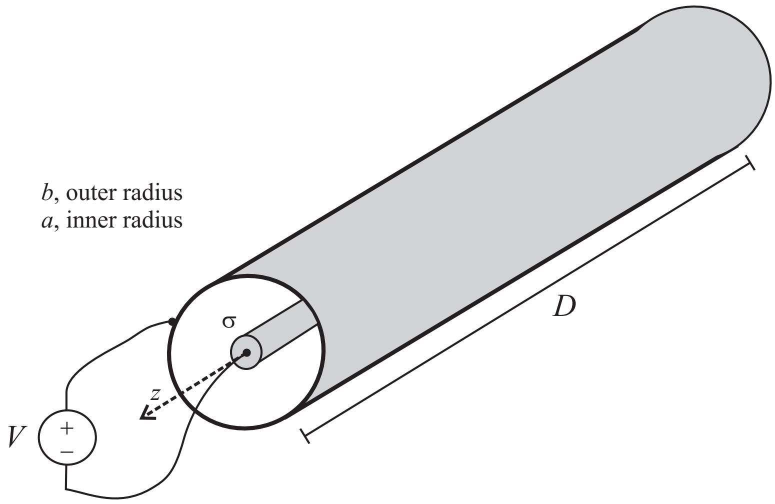
Pledge Signature: ____

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

1. **Coaxial Resistor:** A coaxial structure of length D is filled with a liquid with conductivity σ (Ω^{-1} m^{-1}) and held at voltage V by the source depicted below. The electric field inside the coaxial structure is given by

$$\vec{E}(\rho, \phi, z) = -\frac{K_0}{2\pi\rho\sigma}\hat{\rho}$$

where K_0 is a constant. Answer the following questions based on this scenario. (**50 points**)



(a) What are the units of K_0? (**5 points**)

(b) Solve for the total current I flowing through the coaxial device in terms of the given problem variables. (**15 points**)

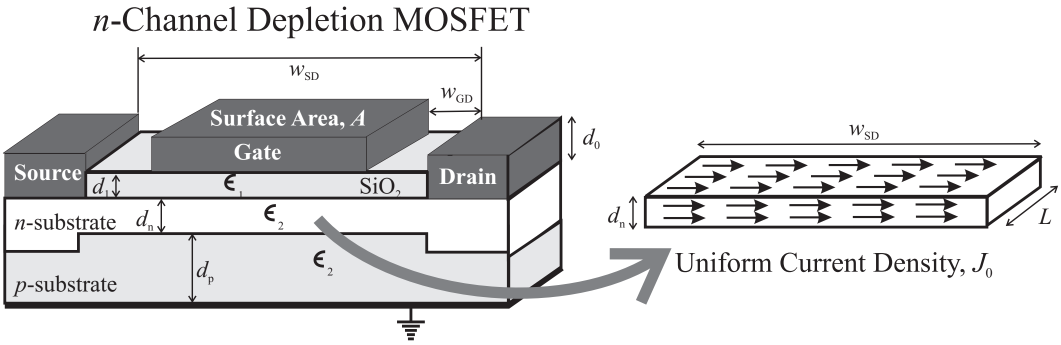
(c) What is the total resistance R of this device in terms of the given problem variables. (**15 points**)

(d) A hole is drilled in the coaxial structure and the liquid is completely drained from the coaxial structure, filling the cavity with air. At steady state, how much charge is deposited on the outer conductor in terms of the given problem variables. (**10 points**)

(e) In the previous question, did the material drain out slowly with the help of gravity or did it violently squirt out? Explain your reasoning. (**5 points**)

2. **Simple Generator:** A square loop of conductive wire with total electrical resistance R and side length D rotates about the y-axis in a constant magnetic flux density of $B_0\hat{z}$ at a speed of f rotations per second. Write a mathematical expression for $I(t)$ around this wire in terms of the problem variables. (**15 points**)

3. **MOSFET Current:** The current between the source and drain of a depletion MOSFET can be crudely modeled as a rectangular prism of constant current density as shown below. As an electromagnetics engineer for RFMP (ridiculously fast micro-processors) Inc, you must estimate the magnetic fields around this circuit as a key part of determining parasitic inductances of the device. (**35 points**)



(a) Write an expression for the total current, I, through the MOSFET. (**5 points**)

(b) In the space below, clearly sketch and label the geometry and orientation for the current density that you have chosen to evaluate $\vec{H}(x, y, z)$. (**5 points**)

(c) Based on your answer in (a), set up and simplify the integral to solve for $\vec{H}(x, y, z)$ using the Biot-Savart relationship, *but do not evaluate it.* (**20 points**)

(d) Why is it incomplete to model magnetic field solely due to this current sheet? (**5 points**)

Formula Sheet

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon r^2}\hat{r} = \int\limits_L \frac{\rho_L(\vec{r}')(\vec{r}-\vec{r}')dL}{4\pi\epsilon||\vec{r}-\vec{r}'||^3} = \iint\limits_S \frac{\rho_S(\vec{r}')(\vec{r}-\vec{r}')dS}{4\pi\epsilon||\vec{r}-\vec{r}'||^3} = \iiint\limits_V \frac{\rho_V(\vec{r}')(\vec{r}-\vec{r}')dV}{4\pi\epsilon||\vec{r}-\vec{r}'||^3}$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon r} = \int\limits_L \frac{\rho_L(\vec{r}')dL}{4\pi\epsilon||\vec{r}-\vec{r}'||} = \iint\limits_S \frac{\rho_S(\vec{r}')dS}{4\pi\epsilon||\vec{r}-\vec{r}'||} = \iiint\limits_V \frac{\rho_V(\vec{r}')dV}{4\pi\epsilon||\vec{r}-\vec{r}'||}$$

$$\vec{H}(\vec{r}) = \int\limits_L \frac{Id\vec{L}\times(\vec{r}-\vec{r}')}{4\pi||\vec{r}-\vec{r}'||^3} = \iint\limits_S \frac{\vec{K}(\vec{r}')\times(\vec{r}-\vec{r}')dS}{4\pi||\vec{r}-\vec{r}'||^3} = \iiint\limits_V \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')dV}{4\pi||\vec{r}-\vec{r}'||^3}$$

$$I = \iint\limits_S \vec{J}(\vec{r}')\cdot d\hat{n} \qquad \Phi_M = \iint\limits_S \vec{B}(\vec{r}')\cdot d\hat{n} \qquad V = -\frac{d\Phi_M}{dt}$$

Variable of integration: $\vec{r}'$ Point of observation: $\vec{r}$

$$\vec{E} = -\nabla V \qquad V = \int \vec{E}\cdot d\vec{L} \qquad \vec{J} = \sigma\vec{E}\,(\sigma \text{ units of S/m})$$

$$\nabla^2 V = \nabla\cdot\nabla V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \qquad \nabla\times\vec{E} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{D} = \epsilon\vec{E} \qquad \vec{B} = \mu\vec{H} \qquad \nabla\cdot\vec{D} = \rho_v \qquad \vec{J} = \sigma\vec{E} \qquad V = IR$$

Capacitance: $C = \frac{Q}{V}$ Coaxial Transmission Line (F/m): $C = \frac{\epsilon}{2\pi\ln(b/a)}$ Cap Energy: $\frac{1}{2}CV^2$

Stored Field Energy: $W = \frac{1}{2}\iiint\limits_V \left[\vec{B}\cdot\vec{H} + \vec{D}\cdot\vec{E}\right] dv$

DIVERGENCE

CARTESIAN $\nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

CYLINDRICAL $\nabla \cdot \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

SPHERICAL $\nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$

GRADIENT

CARTESIAN $\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$

CYLINDRICAL $\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$

SPHERICAL $\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$

CURL

CARTESIAN $\nabla \times \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z$

CYLINDRICAL $\nabla \times \mathbf{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right]\mathbf{a}_z$

SPHERICAL $\nabla \times \mathbf{H} = \frac{1}{r\sin\theta}\left[\frac{\partial(H_\phi \sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial(rH_\phi)}{\partial r}\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right]\mathbf{a}_\phi$

LAPLACIAN

CARTESIAN $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

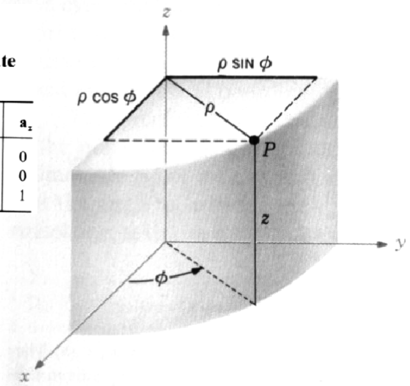
CYLINDRICAL $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

SPHERICAL $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$

Cylindrical Coordinates

Dot products of unit vectors in cylindrical and cartesian coordinate systems

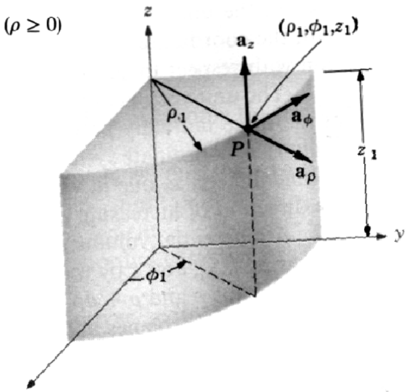
	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1



$\rho = \sqrt{x^2 + y^2} \qquad (\rho \geq 0)$

$\phi = \tan^{-1}\frac{y}{x}$

$z = z$

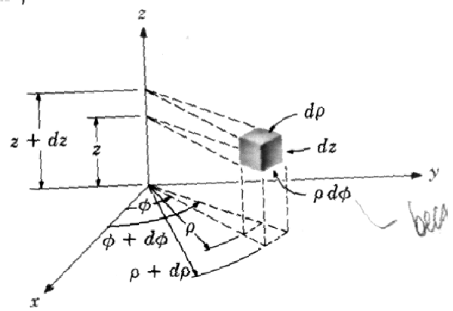


(b)

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

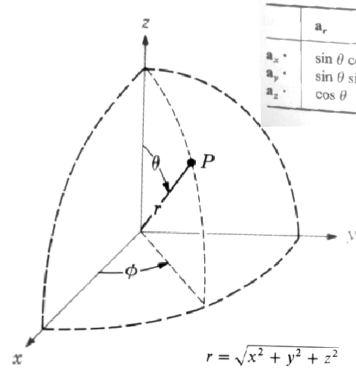


(c)

Spherical Coordinates

Dot products of unit vectors in spherical and cartesian coordinate systems

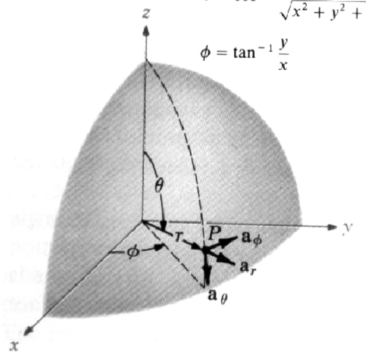
	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0



$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$

$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$

$\phi = \tan^{-1}\frac{y}{x}$



$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$