ECE 3025: Electromagnetics

Solutions to TEST 1 (~~Spring 2008~~)
Fall 2009

(1) **Short Answer Section**

(a) a) coplanar strip, b) microstrip, c) symmetrical stripline, d) coaxial cable, e) parallel plate

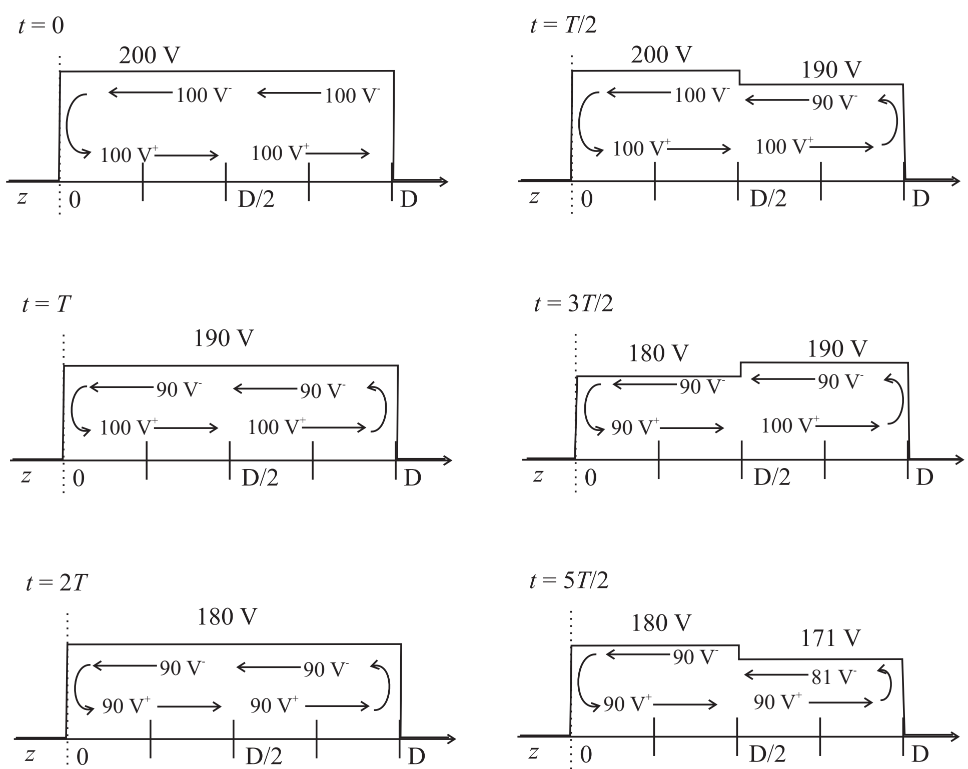
(b) $Z_L = Z_0/N$, $\Gamma = \frac{1-N}{1+N}$

(c) $Z_L = NZ_0$, $\Gamma = \frac{N-1}{N+1}$

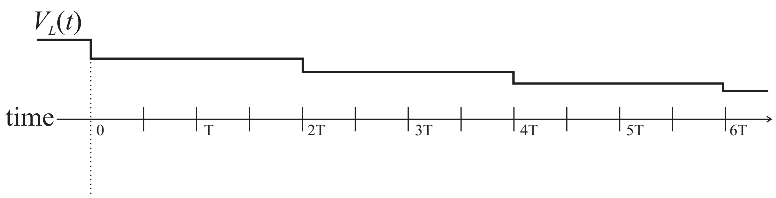
(d) False

(e) open circuit

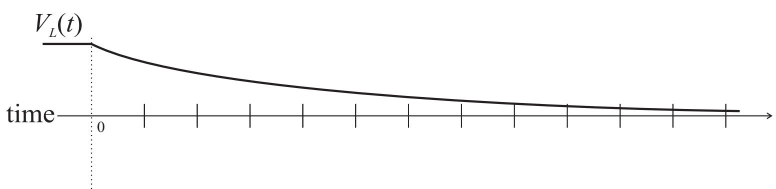
(2) **Discharge of a Long Capacitor:** This problem is identical to the homework problem, only now the long, skinny capacitor is mismatched to the finger, with a reflection coefficient of $\Gamma_L = 0.9$. As such, it will take a lot longer than $2T$ to discharge because a diminishing voltage waveform will continue to circulate around the line, gradually stepping down the total load voltage:



(a) Each step-down in voltage occurs during even multiples of a transit time:



(b) A charged, lump-sum capacitor has simply an exponential discharge across the resistive finger:



(c) The graphs in part (a) will resemble the graph in part (b) if either of the following conditions are met: the capacitor is relatively short (small T) or the line is much more capacitive than inductive (small Z_0). Either of these conditions will cause the steps in part (a) to appear small and/or drawn out in time, so that it will simply look like a granular version of exponential decay.

(d) (**Bonus +5 Points**): This was a tough one, but a great conceptual exercise to recognize that the RC-characteristic you studied in circuits class is, under certain conditions, the same solution that we study in this class! The lumped parameter RC circuit discharges with the following characteristic for $t \geq 0$:

$$V(t) = V_0 \exp\left(-\frac{t}{R_L C'}\right) = V_0 \exp\left(-\frac{t}{R_L C D}\right)$$

where C' is the total capacitance, which is related to the per-unit-length capacitance, C ($C' = CD$). In the transmission line model, there is a geometrically step-decaying solution which can be expressed as

$$V(t) = V_0/2 \underbrace{\left(1 + \frac{R_L - Z_0}{R_L + Z_0}\right)}_{\tau_L} \underbrace{\left(\frac{R_L - Z_0}{R_L + Z_0}\right)}_{\Gamma_L}^{\lfloor \frac{t}{2T} \rfloor}$$

where $\lfloor \cdot \rfloor$ denotes the floor (round down) operation, ensuring that another reflection coefficient is added to the expression every $2T$ to achieve that stair-stepped decay. If the skinny capacitor has a small transmit time, then we can approximate this voltage as

$$V(t) \approx V_0/2 \left(1 + \frac{R_L - Z_0}{R_L + Z_0}\right) \left(\frac{R_L - Z_0}{R_L + Z_0}\right)^{\frac{t}{2DZ_0C}}$$

which takes use of the relationship $T = D\sqrt{LC} = DZ_0C$. Now, if the line is much more capacitive than inductive, $Z_0 = \sqrt{\frac{L}{C}}$ becomes very small. Let's take that limit:

$$V(t) \approx V_0/2 \underbrace{\left(1 + \frac{R_L - Z_0}{R_L + Z_0}\right)}_{\lim\limits_{Z_0\to 0}=2} \left[\underbrace{\left(\frac{R_L - Z_0}{R_L + Z_0}\right)^{\frac{1}{Z_0}}}_{\lim\limits_{Z_0\to 0}=\exp\left(-\frac{2}{R_L}\right)}\right]^{\frac{t}{2DC}} \approx V_0 \exp\left(-\frac{t}{R_L DC}\right)$$

It's our original RC-constant solution for a lumped-sum capacitor! Under conditions of small Z_0 and/or small T, these graphs would look identical on an oscilloscope!

(3) **T-line Sequence Problem:** This problem is the "anti-death star": a topologically simple transmission line problem. The line is connected back in on itself, but this in no way changes our analysis. If you don't panic and put the right equivalent transmission line models into the circuit, the following results are achieved for V^+:

State 0 0 V; system is uncharged.

State 1 4 V; a forward and a backward-traveling wave of 4 V are launched simultaneously from both ends of the line.

State 2 6 V; steady-state circuit shows effectively an open circuit voltage of 12 V and both ends and no current flowing into or out of the line. $V^+ = (V_L + I_L Z_0)/2 = 6$ V, which is also V^-.

State 3 6 V; the system is in equilibrium, current still does not (cannot) flow between transmission line ports.

State 4 6 V; this state persists as there is no resistor to discharge across, no connection from top plate to bottom. Looks like a parallel plate capacitor.

State 5 2 V; there is now a load voltage of 8 V and a current $I_L = 4/Z_0$ flowing across the terminals of both sides of the transmission line. On the load side, $V^+ = (V_L + I_L Z_0)/2 = 6$ V and $V^- = (V_L - I_L Z_0)/2 = 2$ V. This behavior is mirrored on the source side, except V^+ and V^- are flipped.

State 6 0 V; completely discharged.

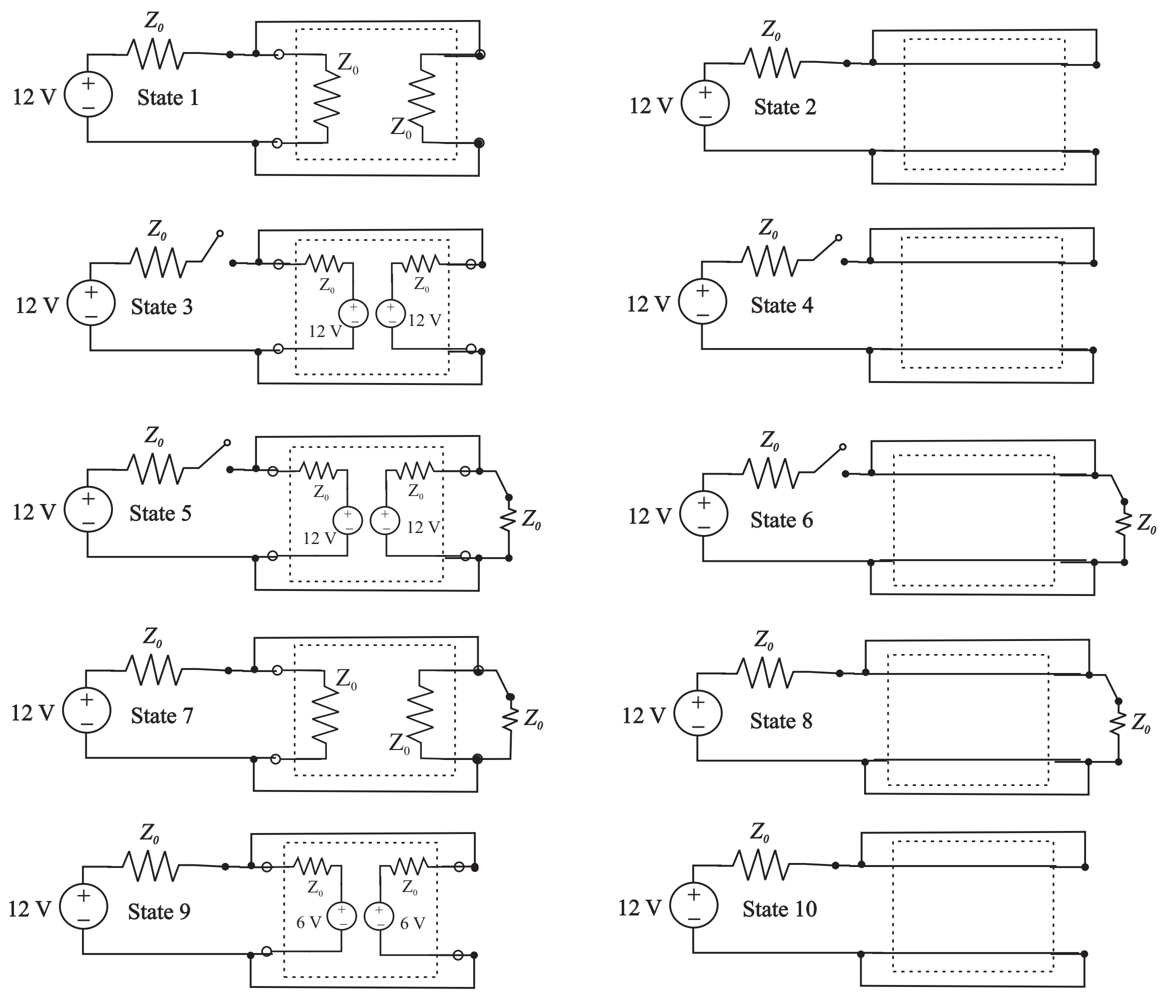
State 7 3 V; we can use the discharged model for the transmission line. A forward and a backward-traveling wave of 3 V are launched simultaneously from both ends of the line.

State 8 3 V; the forward and backward propagating waves are identical at steady-state and must add up to 6 V.

State 9 5 V; there is a total of 8 V across both ends of the line and a current of $-2/Z_0$ flowing across both the terminals of the line. On the load side, $V^+ = (V_L + I_L Z_0)/2 = 3$ V and $V^- = (V_L - I_L Z_0)/2 = 5$ V. This behavior is mirrored on the source side, except V^+ and V^- are flipped.

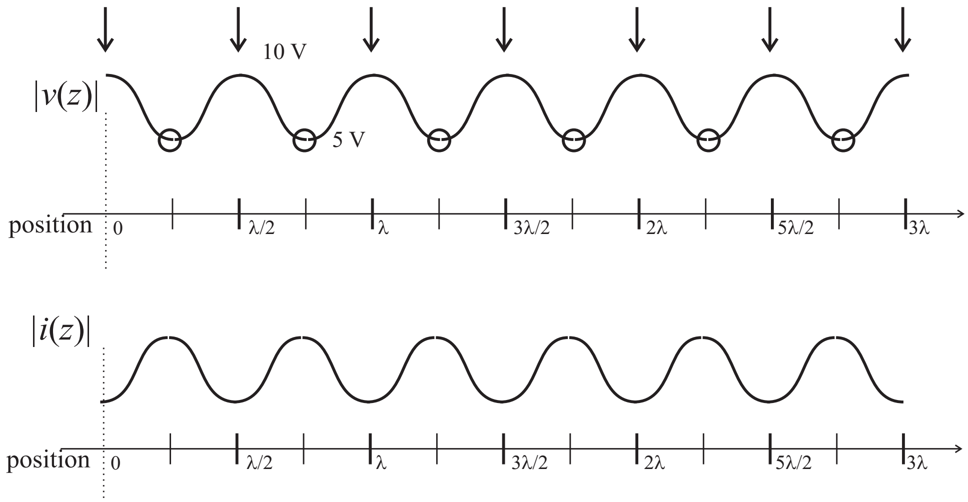
State 10 4 V; identical to state 2.

The equivalent circuits used for this problem are shown below:



(4) **Sinusoidal T-lines:** Answer the following questions based on time-harmonic estimation of a transmission line connected to a load R_L.

(a) For VSWR of 2, the peak total voltage will be 10 V and the minimum voltage will be 5 V, with peaks and nulls alternating every $lambda/4$:



(b) Thevenin equivalent impedance is defined as $v(z)/i(z)$ at a given point; see above for labeling.

(c) $R_L = 2Z_0$ results in a reflection coefficient of $+1/3$ and a VSWR of 2. Note that you could also achieve VSWR of 2 with a purely resistive $R_L = Z_0/2$, but this violates the condition of $R_L > Z_0$ given in the problem statement.

(d) At steady-state time-harmonic excitation, purely inductive loads result in VSWR of ∞, so clearly VSWR would increase.