ECE 3025: Electromagnetics

SOLUTIONS TO TEST 2 (Fall 2003)

(1) **Short Answer Section**

(a) $\frac{Z_0^2}{Z_L}$

(b) dB/m

(c) $\hat{x} \times \hat{y} = \hat{z}$

(d) $\frac{1}{2\pi}$ m

(e) steady

(f) false

(g) 0

(h) parallel/colinear/pointing in the same direction

(i) $7\exp(-j\frac{\pi}{4})$ (1) 7 (2)

(j) increases (1) skin (2)

(2) **Descriptive Answer Section** (20 points)

(a) The kind caused a perfect reflection midway down the line. The peaks of the bulge occur every $\lambda/2$ or 0.5m.

(b) $\hat{a}_r = \hat{a}_z$, $\hat{a}_\theta = \hat{a}_x$, $\vec{a}_\phi = \hat{a}_y$

(3) **Transmission Line with Sinusoidal Excitation:**

(a) In the equations above, circle the portion of the solution representing the backward-propagating current waveform. *The term* $\frac{1}{2}\exp(j\pi + j\beta[z - D])$ *should be circled.*

(b) In the equations above, box the forward-propagating *amplitude* of the voltage waveform. *The amplitude 100 should be boxed.*

(c) This follows from the phasor inverse transform definition (see formula sheet)

$$\begin{aligned} v(t,0) &= \text{Real}\left\{\tilde{v}(0)\exp(j2\pi ft)\right\} \\ &= 100\cos(2\pi ft + \beta D) + 50\cos(2\pi ft - \beta D + \pi) \\ &= 100\cos(2\pi ft + \beta D) - 50\cos(2\pi ft - \beta D) \end{aligned}$$

Anyone who made it to line two got full credit.

(d) $Z_0 = 100\Omega$

(e) There are two ways of doing this. First, we immediately see that $|\Gamma| = 0.5$ because the reflected voltage/current wave is half the magnitude of the forward voltage/current wave. This can be plugged directly into one of the VSWR formulas to get VSWR=3. The same result can be obtained from the following reasoning: VSWR is the ratio of max voltage to min voltage on the line; the max voltage occurs when the forward wave adds in phase with the backwards wave (100+50 V); the min voltage occurs when the forward wave adds out-of-phase (destructively) with the backwards wave (100-50 V); thus, the VSWR will be 150/50 or 3.

(f) For this problem, I gave full credit to anyone who put down line 1 of the following answer, but did not finish the calculation. Very few people got full credit for this problem.

$$\begin{aligned} Z_L &= \frac{\tilde{v}(D)}{\tilde{i}(D)} \\ &= \frac{100 + 50\exp(j\pi)}{1 - \frac{1}{2}\exp(j\pi)} \\ &= \frac{100 - 50}{1 + \frac{1}{2}} \\ &= \frac{100}{3}\Omega \end{aligned}$$

Many people tried to solve this from the VSWR by using the property $|\Gamma| = \frac{1}{2}$ to backsolve $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$. This actually doesn't work because it only gives you the *magnitude* of the reflection coefficient. Many people worked the problem with $\Gamma = \frac{1}{2}$ and found $Z_L = 300\Omega$, when in fact the reflection coefficient for this particular example was $\Gamma = -\frac{1}{2}$.

(4) **Electron Gun in a TV:**

(a) Below is the full integral setup for the finite line of charge.

$$\vec{E}(\vec{r}) = \int_0^D \frac{\overbrace{\rho_L(\vec{r}')}^{\rho_L}(\vec{r} - \vec{r}')\overbrace{dL}^{dz'}}{4\pi\epsilon|\vec{r} - \vec{r}'|^3}$$

$$\vec{E}(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z) = \int_0^D \frac{\rho_L(\overbrace{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}^{\vec{r}} \overbrace{-0\hat{a}_x - 0\hat{a}_y - z'\hat{a}_z}^{-\vec{r}'})dz'}{4\pi\epsilon|\underbrace{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}_{\vec{r}} \underbrace{-0\hat{a}_x - 0\hat{a}_y - z'\hat{a}_z}_{-\vec{r}'}|^3}$$

$$\begin{aligned} \vec{E}(x, y, z) &= \int_0^D \frac{\rho_L(x\hat{a}_x + y\hat{a}_y + [z - z']\hat{a}_z)dz'}{4\pi\epsilon(x^2 + y^2 + [z - z']^2)^{\frac{3}{2}}} \\ &= (x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)\frac{\rho_L}{4\pi\epsilon}\int_0^D \frac{dz'}{(x^2 + y^2 + [z - z']^2)^{\frac{3}{2}}} - \hat{a}_z\frac{\rho_L}{4\pi\epsilon}\int_0^D \frac{z'dz'}{(x^2 + y^2 + [z - z']^2)^{\frac{3}{2}}} \end{aligned}$$

Nearly full credit was given to students who made it to line 1. Complete credit was given for anyone who showed the explicit dependence of $\vec{r}'$ and the variable of integration z, which occurs in line 2.

(b) This follows directly from the formula on the back sheet for (approximately) infinite line charge.

$$\vec{F}(\rho) = q\vec{E} = \frac{q\rho_L}{2\pi\rho\epsilon}\hat{a}_\rho$$

where $q = -1.60 \times 10^{-19}$. Since both q and ρ_L are negative, the force will be away from the line of charge.

(c) The arrow should point in the -z direction (left). If you reversed the arrow (sign error) you received -4. If your arrow was pointing in any other direction or was not placed in the box labeled electron gun, you received 0.