Solution to Practice Questions 2

ECE 3025: Electromagnetics

(1) **Short Answer Section**

(a) $\frac{2\pi v_p}{\lambda}$

(b) one-half

(c) matched

(d) $\frac{11}{2}$, $\frac{9}{2}$ V

(e) true

(f) true

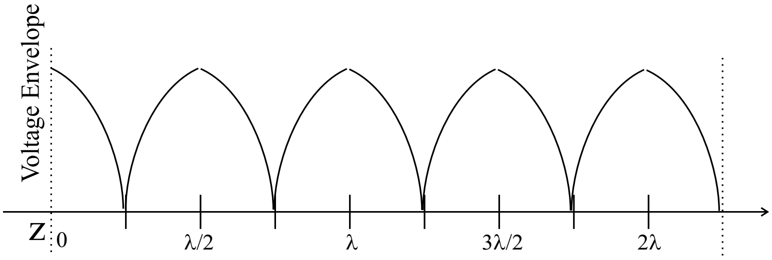
(g) orthogonal

(2) **Descriptive Answer Section**

(a) **Phasor Quantities:** The function $v(z,t)$ is the physical voltage on the transmission line as a function of position, z, and time, t. The phasor voltage $\tilde{v}(z)$ represents the amplitudes and phases of sinusoidal voltages on the transmission lines as a function of position, z; the time-dependence, t, has been removed since the excitation is assumed to be time-harmonic. The envelope, $R(z)$ is equal to the amplitude of the sinusoids on the transmission line.

(b) **Leaky-Feeder Line:** Although it may sound counter-intuitive, a uniformly punctured line will produce leaky-feeder lines with non-uniform coverage. Why? Because constant loss in a transmission line introduces an exponential taper on the signal as it travels down the line. In order to make a leaky-feeder line with uniform radiation loss, we must increase the loss as a function of line length, to compensate for the fact that the signal is losing power and cannot shed as much power with the same attenuation constant.

(c) **Superconductors for Power Storage:**



(d) **Parallel-Piped:** Volume is equal to

$$|\vec{A} \cdot \left(\vec{B} \times \vec{C}\right)|$$

or

$$|\vec{B} \cdot \left(\vec{A} \times \vec{C}\right)|$$

or

$$|\vec{C} \cdot \left(\vec{B} \times \vec{A}\right)|$$

Work-out Problem Section

(3) **Fields in Ionosphere:** The total charge in the ionosphere can be calculated using the following integral:

$$Q = \int_{6080}^{6180} \int_{0}^{2\pi} \int_{0}^{\pi} \left[-10^{-10} \sin(\theta)\right] \underbrace{r^2 \sin(\theta)\, dr\, d\phi\, d\theta}_{dV} = -2\pi \times 10^{-10} \int_{6080}^{6180} \int_{0}^{\pi} r^2 \sin^2(\theta)\, dr\, d\theta$$

This can be simplified more, but on a 50-minute test there is no need to take it any further. For the field in outer space due to this charge in (b), we (approximately) use the Coulomb point charge equation:

$$\vec{E}(\vec{r}) = \frac{Q\vec{r}}{4\pi\epsilon_0 |\vec{r}|^3} \quad \text{or} \quad \vec{E}(\vec{r}) = \frac{Q\hat{a}_r}{4\pi\epsilon_0 |\vec{r}|^2}$$

If we wanted to calculate this field exactly, we would use the following integral:

$$\vec{E}(\vec{r}) = \int_{6080}^{6180} \int_{0}^{2\pi} \int_{0}^{\pi} \frac{\left[-10^{-10} \sin(\theta')\right] (\vec{r} - \vec{r}')}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3} \underbrace{r'^2 \sin(\theta')\, dr'\, d\phi'\, d\theta'}_{dV}$$

Note: evaluating these integrals is not important, but it is important to be able to set them up (given the general equations on the formula sheet) and establish the limits of integration.

(4) **Night Vision Goggles:** The total work performed on the electron is

$$\text{Work} = -q \int_{0}^{D} \vec{E}(\vec{r}) \cdot \underbrace{d\vec{L}}_{dx\, \vec{a}_x} = -10,000q \int_{0}^{D} \sin\left(\pi \frac{x}{D}\right) dx = -\frac{20,000Dq}{\pi}$$

where $q = -1.6 \times 10^{-19}$ Coulombs.

(5) **Vector Algebra Proofs:**

(a)

$$\begin{aligned}\tan\theta_{AB} &= \frac{\sin\theta_{AB}}{\cos\theta_{AB}} \\ &= \frac{|\vec{a}||\vec{b}|\sin\theta_{AB}}{|\vec{a}\vec{b}|\cos\theta_{AB}} \\ &= \frac{|\vec{a}\times\vec{b}|}{\vec{a}\cdot\vec{b}}\end{aligned}$$

(b)

$$\begin{aligned}\vec{a}\cdot(\vec{a}\times\vec{b}) &= \vec{a}\cdot(\hat{a}_N|\vec{a}||\vec{b}|\sin\theta_{AB}) \\ &= \underbrace{\vec{a}\cdot\hat{a}_N}_{0}|\vec{a}||\vec{b}|\sin\theta_{AB} \\ &= 0\end{aligned}$$

(c)

$$\begin{aligned}\vec{a}\cdot\vec{a} &= a_x^2+a_y^2+a_z^2 \\ &= \sqrt{a_x^2+a_y^2+a_z^2}\sqrt{a_x^2+a_y^2+a_z^2} \\ &= |\vec{a}||\vec{a}| \\ &= |\vec{a}|^2\end{aligned}$$

(d)

$$\begin{aligned}\vec{a}\times\vec{b} &= \det\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z\end{vmatrix} \\ &= -\det\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z \\ b_x & b_y & b_z \\ a_x & a_y & a_z\end{vmatrix} \\ &= -\vec{b}\times\vec{a}\end{aligned}$$