ECE 3025: Electromagnetics

Solutions to TEST 2 (Fall 2005)

(1) **Short Answer Section**

(a) point, line, surface, and volume charges

(b) 1

(c) streamlines

(d) Nepers/m, dB/m

(e) $3Z_0$, $\frac{Z_0}{3}$

(2) **Sinusoidal Transmission Lines:**

The $j100\,\Omega$ load-transforms to the source side as a $-j100\,\Omega$ load, which is the Thevenin equivalent circuit of the transmission line. Hooked directly to the sinusoidal source, we can calculate that the input voltage is going to be $200\angle 0°$ V and the input current is going to be $j2$ A. Now write down the general phasor solution to the transmission line equations:

$$\begin{aligned} \tilde{v}(z) &= V^+ \exp(-j\beta z) + V^- \exp(+j\beta z) & \tilde{v}(0) &= V^+ + V^- &= 200 \text{ V} \\ \tilde{i}(z) &= \frac{V^+}{Z_0} \exp(-j\beta z) - \frac{V^-}{Z_0} \exp(+j\beta z) & \tilde{i}(0) &= \frac{V^+}{Z_0} - \frac{V^-}{Z_0} &= j2 \text{ A} \end{aligned}$$

As shown above, we evaluate for current and voltage at the front of the line and equate this to our input voltage and current. This gives us two equations and two unknowns. Solving for V^+ and V^-, we get the following solution:

$$\begin{aligned} \tilde{v}(z) &= (100 + j100) \exp(-j\beta z) &+& \quad (100 - j100) \exp(+j\beta z) \\ \tilde{i}(z) &= (1 + j) \exp(-j\beta z) &-& \quad (1 - j) \exp(+j\beta z) \end{aligned}$$

(3) **Transmission Line Solutions:**

Turns out these are *all* invalid. Writing *valid* results in -5 points. Writing *invalid* for the wrong reason results in -3 points.

(a)
$$
\begin{aligned}
v(z,t) &= 100\cos(2\pi ft - z) + 50\sin(2\pi ft + z) \\
i(z,t) &= 5\cos(2\pi ft - z) - 5\sin(2\pi ft + z)
\end{aligned}
$$

It would appear that this line has 20Ω of intrinsic impedance in the forward direction and a contradictory 10Ω impedance in the backward direction.

(b)
$$
\begin{aligned}
\tilde{v}(z) &= 200\exp\left(-j80z\right) \\
\tilde{i}(z) &= (5-3j)\exp\left(-j80z\right)
\end{aligned}
$$

The impedance is complex, implying that this is a lossy transmission line. So where is the attenuation constant?

(c)
$$
\begin{aligned}
\tilde{v}(z) &= 77\exp\left(-[7+5j]z\right) + 55\exp\left(-[7-5j]z\right) \\
\tilde{i}(z) &= \frac{77}{Z_0}\exp\left(-[7+5j]z\right) - \frac{55}{Z_0}\exp\left(-[7-5j]z\right)
\end{aligned}
$$

As some astute students noticed, the attenuation constant has the wrong sign in this solution for the backwards-traveling wave – the wave *grows* in amplitude instead of decays!

(d)
$$
\begin{aligned}
v(z,t) &= 75\exp\left(-[t-z]^2\right) + 75\exp\left(-[t+\frac{z}{2}]^2\right) \\
i(z,t) &= \exp\left(-[t-z]^2\right) - \exp\left(-[t+\frac{z}{2}]^2\right)
\end{aligned}
$$

A hold-over from pure time domain pulses. The velocity of propagation is twice as fast for the backwards-traveling wave when compared to the forwards-traveling wave. Impossible. The fact that the pulse is shaped like a Gaussian does not contradict anything.

(4) **Conical Charge Dipole:**

Lots of partial credit given for this one! You could save yourself a lot of trouble if you recognize that the electric field components in $\hat{x}$ and $\hat{y}$ due to the positive charges on the top cone will perfectly cancel out with the corresponding components due to the negative charges on the bottom cone. If you did not see this symmetry, you could still discover this cancellation with the standard technique for setting these problems up. The full-length solution is given below:

$$\vec{E}(\vec{r}) = \int_{S} \frac{\overbrace{\rho_S(\vec{r}')}^{\rho_0}(\vec{r}-\vec{r}')\,dS}{4\pi\epsilon||\vec{r}-\vec{r}'||^3}$$

$$\vec{E}(x,y,0) = \frac{\rho_0}{4\pi\epsilon}\int_{\substack{\text{Top}\\\text{Cone}}} \frac{(\overbrace{x\hat{x}+y\hat{y}+0\hat{z}}^{\vec{r}}\overbrace{-x'\hat{x}-y'\hat{y}-z'\hat{z}}^{-\vec{r}'})\,dS}{||\underbrace{x\hat{x}+y\hat{y}+0\hat{z}}_{\vec{r}}\underbrace{-x'\hat{x}-y'\hat{y}-z'\hat{z}}_{-\vec{r}'}||^3} - \frac{\rho_0}{4\pi\epsilon}\int_{\substack{\text{Bot.}\\\text{Cone}}} \frac{(\overbrace{x\hat{x}+y\hat{y}+0\hat{z}}^{\vec{r}}\overbrace{-x'\hat{x}-y'\hat{y}-z'\hat{z}}^{-\vec{r}'})\,dS}{||\underbrace{x\hat{x}+y\hat{y}+0\hat{z}}_{\vec{r}}\underbrace{-x'\hat{x}-y'\hat{y}-z'\hat{z}}_{-\vec{r}'}||^3}$$

$$= \frac{\rho_0}{4\pi\epsilon}\int_0^{\frac{1}{2}}\int_0^{2\pi} \frac{[(x-r\sin 45^\circ\cos\phi)\hat{x} + (y-r\sin 45^\circ\sin\phi)\hat{y} - r\cos 45^\circ\hat{z}]}{[(x-r\sin 45^\circ\cos\phi)^2 + (y-r\sin 45^\circ\sin\phi)^2 + r^2\cos^2 45^\circ]^{\frac{3}{2}}} r\sin 45^\circ\, d\phi\, dr$$

$$- \frac{\rho_0}{4\pi\epsilon}\int_0^{\frac{1}{2}}\int_0^{2\pi} \frac{[(x-r\sin 135^\circ\cos\phi)\hat{x} + (y-r\sin 135^\circ\sin\phi)\hat{y} - r\cos 135^\circ\hat{z}]}{[(x-r\sin 135^\circ\cos\phi)^2 + (y-r\sin 135^\circ\sin\phi)^2 + r^2\cos^2 135^\circ]^{\frac{3}{2}}} r\sin 135^\circ\, d\phi\, dr$$

$$= \frac{\rho_0}{4\sqrt{2}\pi\epsilon}\int_0^{\frac{1}{2}}\int_0^{2\pi} \frac{\left[(x-r\frac{1}{\sqrt{2}}\cos\phi)\hat{x} + (y-r\frac{1}{\sqrt{2}}\sin\phi)\hat{y} - r\frac{1}{\sqrt{2}}\hat{z}\right]}{\left[(x-r\frac{1}{\sqrt{2}}\cos\phi)^2 + (y-r\frac{1}{\sqrt{2}}\sin\phi)^2 + \frac{r^2}{2}\right]^{\frac{3}{2}}} r\, d\phi\, dr$$

$$- \frac{\rho_0}{4\sqrt{2}\pi\epsilon}\int_0^{\frac{1}{2}}\int_0^{2\pi} \frac{\left[(x-r\frac{1}{\sqrt{2}}\cos\phi)\hat{x} + (y-r\frac{1}{\sqrt{2}}\sin\phi)\hat{y} + r\frac{1}{\sqrt{2}}\hat{z}\right]}{\left[(x-r\frac{1}{\sqrt{2}}\cos\phi)^2 + (y-r\frac{1}{\sqrt{2}}\sin\phi)^2 + \frac{r^2}{2}\right]^{\frac{3}{2}}} r\, d\phi\, dr$$

$$= -\frac{\rho_0\hat{z}}{4\pi\epsilon}\int_0^{\frac{1}{2}} r^2\, dr \int_0^{2\pi} d\phi \frac{1}{\left[(x-\frac{r}{\sqrt{2}}\cos\phi)^2 + (y-\frac{r}{\sqrt{2}}\sin\phi)^2 + \frac{r^2}{2}\right]^{\frac{3}{2}}}$$

* Most of the credit for this problem was given for reaching this step.