ECE 3025: Electromagnetics

Solutions to TEST 2 (Fall 2006)

(1) **Short Answer Section**

(a) dot product

(b) conductor (1) dielectric (2)

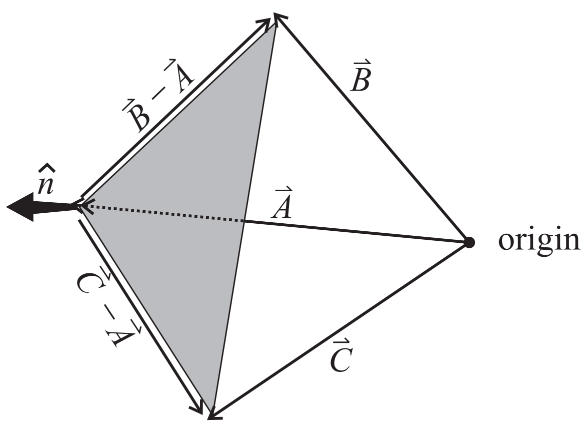
(c) shunt conductance (1) series resistance (2)

(2) **Vector Math:**

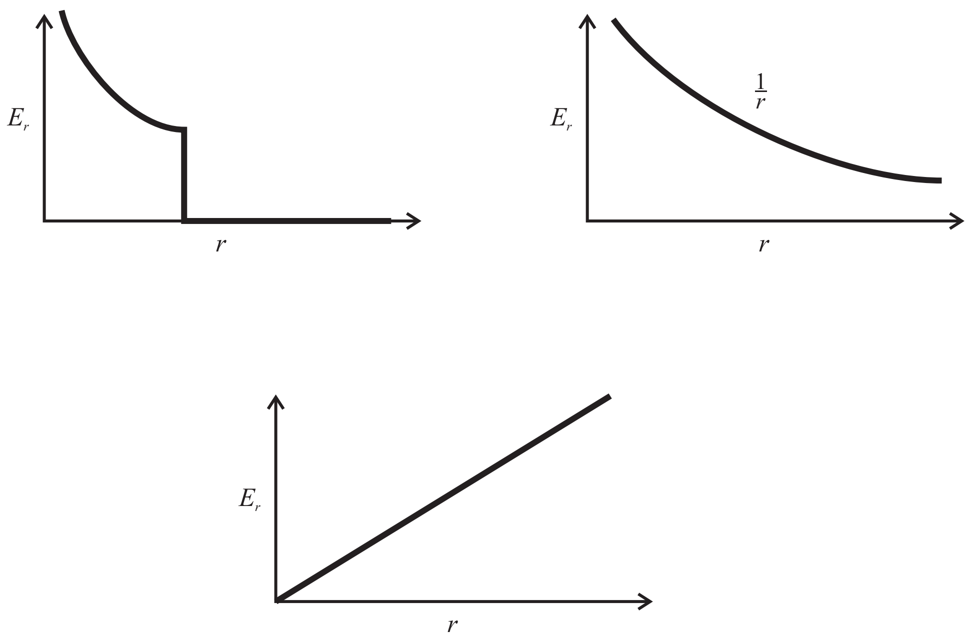
The answer is given below

$$\hat{n} = \frac{(\vec{B} - \vec{A}) \times (\vec{C} - \vec{A})}{\|(\vec{B} - \vec{A}) \times (\vec{C} - \vec{A})\|}$$

Of course, there are some equally correct variants of this equation that may be arrived at through swapping the order and placement. See the diagram below for a visual explanation of this equation.



(3) **Charge Distributions:**



(4) **Night Vision Goggles:**

(a) The arrow points in the $-\hat{x}$ direction.

(b) The minimum voltage is

$$V > \frac{W_0}{-q}$$

(c) The field constant must be

$$E_0 = \frac{W_0}{-qD}$$

(5) **Charge Disks:**

Lots of partial credit given for this one! You could save yourself a lot of trouble if you recognize that the electric field components in $\hat{x}$ and $\hat{y}$ due to the positive charges on the top disk will perfectly cancel out with the corresponding components due to the negative charges on the bottom disk. If you did not see this symmetry, you could still discover this cancellation with the standard technique for setting these problems up. The full-length solution is given below:

$$\vec{E}(\vec{r}) = \int\limits_{\mathrm{S}} \frac{\overbrace{\rho_S(\vec{r}')}^{\rho_S}(\vec{r}-\vec{r}')\,dS}{4\pi\epsilon||\vec{r}-\vec{r}'||^3}$$

$$\vec{E}(0,0,z) = \frac{\rho_S}{4\pi\epsilon}\int\limits_{\substack{\text{Top}\\\text{Disk}}} \frac{(\overbrace{0\hat{x}+0\hat{y}+z\hat{z}}^{\vec{r}}\overbrace{-x'\hat{x}-y'\hat{y}-z'\hat{z}}^{-\vec{r}'})\,dS}{||\underbrace{0\hat{x}+0\hat{y}+z\hat{z}}_{\vec{r}}\underbrace{-x'\hat{x}-y'\hat{y}-z'\hat{z}}_{-\vec{r}'}||^3} - \frac{\rho_S}{4\pi\epsilon}\int\limits_{\substack{\text{Bot.}\\\text{Disk}}} \frac{(\overbrace{0\hat{x}+0\hat{y}+z\hat{z}}^{\vec{r}}\overbrace{-x'\hat{x}-y'\hat{y}-z'\hat{z}}^{-\vec{r}'})\,dS}{||\underbrace{0\hat{x}+0\hat{y}+z\hat{z}}_{\vec{r}}\underbrace{-x'\hat{x}-y'\hat{y}-z'\hat{z}}_{-\vec{r}'}||^3}$$

$$= \frac{\rho_S}{4\pi\epsilon}\int_a^b\int_0^{2\pi} \frac{\overbrace{(-x')\hat{x}+(-y')\hat{y}}^{\text{symmetry}\to 0}+(z-\frac{D}{2})\hat{z}}{\left[\underbrace{(x')^2+(y')^2}_{\rho'^2}+(z-\frac{D}{2})^2\right]^{\frac{3}{2}}}\rho'\,d\phi'\,d\rho' - \frac{\rho_S}{4\pi\epsilon}\int_a^b\int_0^{2\pi} \frac{\overbrace{(-x')\hat{x}+(-y')\hat{y}}^{\text{symmetry}\to 0}+(z+\frac{D}{2})\hat{z}}{\left[\underbrace{(x')^2+(y')^2}_{\rho'^2}+(z+\frac{D}{2})^2\right]^{\frac{3}{2}}}\rho'\,d\phi'\,d\rho'$$

$$= \frac{\rho_S\hat{z}}{4\pi\epsilon}\left(\int_a^b\int_0^{2\pi} \frac{(z-\frac{D}{2})\rho'\,d\phi'\,d\rho'}{\left[\rho'^2+(z-\frac{D}{2})^2\right]^{\frac{3}{2}}} - \int_a^b\int_0^{2\pi} \frac{(z+\frac{D}{2})\rho'\,d\phi'\,d\rho'}{\left[\rho'^2+(z+\frac{D}{2})^2\right]^{\frac{3}{2}}}\right)$$

Full credit for this problem was given for reaching this step. The intrepid could continue on for the bonus +5:

$$\vec{E}(0,0,z) = \frac{\rho_S\hat{z}}{2\epsilon}\left(\int_a^b \frac{(z-\frac{D}{2})\rho'\,d\rho'}{\left[\rho'^2+(z-\frac{D}{2})^2\right]^{\frac{3}{2}}} - \int_a^b \frac{(z+\frac{D}{2})\rho'\,d\rho'}{\left[\rho'^2+(z+\frac{D}{2})^2\right]^{\frac{3}{2}}}\right)$$

$$= \frac{\rho_S\hat{z}}{2\epsilon}\left(\frac{(z+\frac{d}{2})}{\sqrt{b^2+(z+\frac{d}{2})^2}} - \frac{(z+\frac{d}{2})}{\sqrt{a^2+(z+\frac{d}{2})^2}} - \frac{(z-\frac{d}{2})}{\sqrt{b^2+(z-\frac{d}{2})^2}} + \frac{(z-\frac{d}{2})}{\sqrt{a^2+(z-\frac{d}{2})^2}}\right)$$