ECE 3025: Electromagnetics

Solutions to TEST 2 (Spring 2008)

(1) **Power Storage:**

The total distance that the wave on the transmission line travels is obviously Tv_p. Therefore, the total attenuation would be $\alpha_d T v_p$ if α_d were measured in dB/m. This total attenuation must be less than 3 dB. Thus, the following condition must hold:

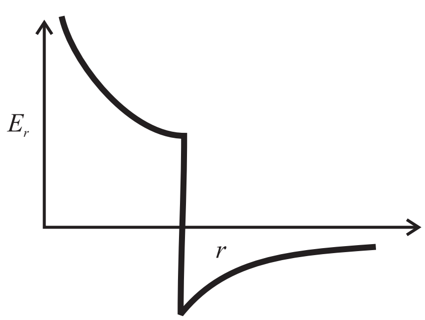
$$\alpha_d < \frac{3000}{T v_p} \,\text{dB/km}$$

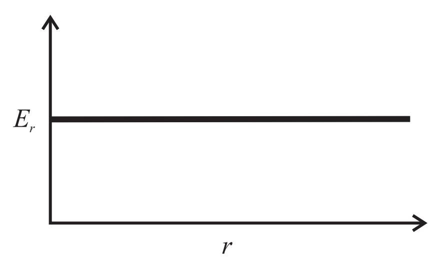
where the extra factor of 1000 is to convert from dB/m to dB/km.

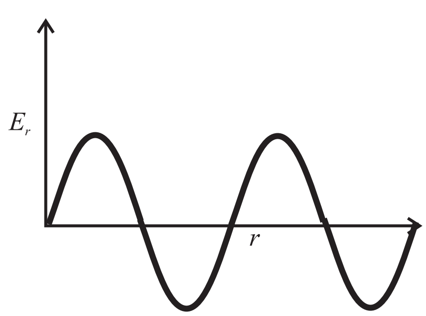
(2) **Vector Math:**

The volume of the parallelpiped is $|\vec{A} \cdot (\vec{B} \times \vec{C})|$. Note that the three vectors could all be commuted in any combination in this expression.

(3) **Charge Distributions:**







(4) **Voltage of a Spiral Charge:**

$$\begin{aligned}
V(\vec{r}) &= \iint_S \frac{\rho_S(\vec{r'})dS}{4\pi\epsilon\|\vec{r}-\vec{r'}\|} \\
&= \iint_S \frac{\rho_o \exp(-\rho')\,dS}{4\pi\epsilon\|(0-x')\hat{x}+(0-y')\hat{y}+(0-0)\hat{z}\|} \\
&= \frac{\rho_o}{4\pi\epsilon}\int_0^\infty \int_{\rho'}^{\rho'+\phi_o} \frac{\exp(-\rho')\,\rho'\,d\rho'\,d\phi'}{\rho'} \\
&= \frac{\rho_o\phi_o}{4\pi\epsilon}\int_0^\infty \exp(-\rho')\,d\rho' \\
&= \frac{\rho_o\phi_o}{4\pi\epsilon}
\end{aligned}$$

(5) **DLP Chip:**

The total electric field is the superposition of the infinite plane charge (a result we know from class and on the formula sheet) and the tilted square:

$$\begin{aligned}
\vec{E}(x,y,z) &= \frac{-\rho_2}{2\epsilon_o}\hat{y} + \iint_S \frac{\rho_S(\vec{r'})(\vec{r}-\vec{r'})dS}{4\pi\epsilon\|\vec{r}-\vec{r'}\|^3} \\
&= \frac{-\rho_2}{2\epsilon_o}\hat{y} + \frac{\rho_1}{4\pi\epsilon_o}\int_0^L dz' \int_{\frac{d}{\sin\phi_o}-\frac{L}{2}}^{\frac{d}{\sin\phi_o}+\frac{L}{2}} d\rho' \, \frac{[(x-\rho'\cos\phi_o)\hat{x}+(y-\rho'\sin\phi_o)\hat{y}+(z-z')\hat{z}]}{[(x-\rho'\cos\phi_o)^2+(y-\rho'\sin\phi_o)^2+(z-z')^2]^{\frac{3}{2}}}
\end{aligned}$$

which is valid for $y > 0$.