ECE 3025: Electromagnetics
Solutions to TEST 2 (Fall 2009)

(1) **Lossy Line:** Voltage magnitude is $V_0 \exp(-\alpha L)$

(2) **Coordinate Systems:**

(a)

$$\hat{\rho} = \hat{y} \qquad \hat{\phi} = -\hat{x} \qquad \hat{z} = \hat{z}$$

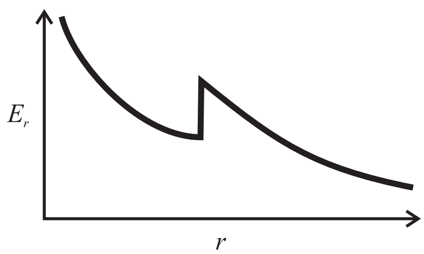
(b)

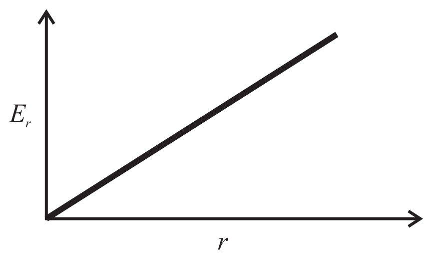
$$\hat{r} = -\hat{x} \qquad \hat{\phi} = -\hat{y} \qquad \hat{\theta} = -\hat{z}$$

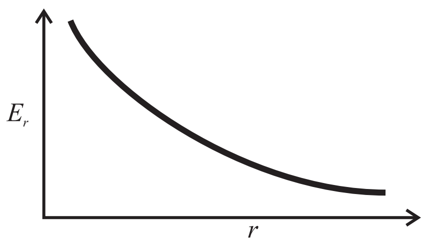
(c)

$$\hat{r} = \hat{z} \qquad \hat{\phi} = \hat{y} \qquad \hat{\theta} = \hat{x}$$

(3) **Electrostatic Charge Distributions:**







(4) **Electrostatic Integrals:**

(a) Integrate over the conical surface charge to find the total charge:

$$Q = \int_0^b dr' \int_0^{2\pi} r' \sin\theta\big|_{\theta=\alpha} \, d\phi' \, \rho_s = \pi \sin\alpha \, b^2 \rho_s$$

(b) The following answer received full credit

$$\vec{E}(0,0,z) = \frac{\rho_s \sin\alpha \, \hat{z}}{4\pi\epsilon} \int_0^b \int_0^{2\pi} \frac{(z - r'\cos\alpha) \, r' dr' \, d\phi'}{\left[r'^2 \sin^2\alpha \cos^2\phi' + r'^2 \sin^2\alpha \sin^2\phi' + (z - r'\cos\alpha)^2\right]^{3/2}}$$

although some astute students were able to further simplify:

$$\vec{E}(0,0,z) = \frac{\rho_s \sin\alpha\,\hat{z}}{2\epsilon} \int_0^b \frac{(z - r' \cos\alpha)\, r' dr'}{\left[r'^2 - 2zr' \cos\alpha + z^2\right]^{3/2}}$$

(c) The reduced, pre-evaluation solution is

$$V(x,y,0) = \frac{\rho_s \sin\alpha\,\hat{z}}{4\pi\epsilon} \int_0^b \int_0^{2\pi} \frac{r' dr'\, d\phi'}{\sqrt{(x - r' \sin\alpha\cos\phi')^2 + (y - r' \sin\alpha \sin\phi') + r'^2 \cos^2\alpha}}$$

A seventh-level coordinate system master could have reduced this solution to the following using cylindrical coordinates, which takes advantage of symmetry in the problem:

$$V(\rho,\phi,0) = \frac{\rho_s \sin\alpha\,\hat{z}}{4\pi\epsilon} \int_0^b \int_0^{2\pi} \frac{r' dr'\, d\phi'}{\sqrt{\rho^2 - 2r'\rho \sin\alpha \sin\phi' + r'^2}}$$

(d) $b \to \infty$, $\alpha \to 90°$

(e) Take the solution for $b = a$ and $\alpha = 0°$ and add it to the solution for $b = a$ and $\alpha = 180°$. Another way would be to take the solution for $b = 2a$ and shift the answer down a units along the z-axis.