Solution to Practice Questions 3

ECE 3025: Electromagnetics

(1) **Short Answer Section**

(a) $\vec{0}$

(b) 0

(c) Stoke's

(d) Faraday's

(e) Ampere's

(f) Coulombs **or** C (1), C/m (2), C/m^2 (3), C/m^3 (4)

(g) Coulombs/s **or** C/s **or** Amps (1), Amps/m (2), Amps/m^2 (3)

(h) $\nabla \times \vec{H} = \vec{J}$

(i) false

(j) Laplacian

(k) scalar, scalar

(l) vector, vector

(m) vector, scalar

(n) scalar, vector

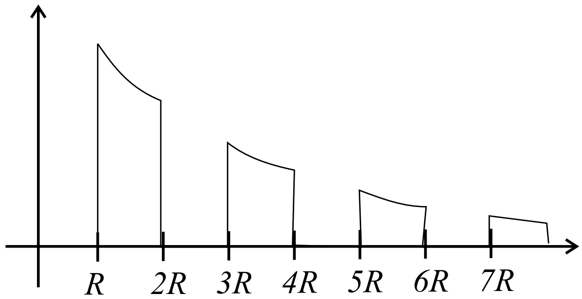
(o) Gauss's

(2) **Descriptive Answer Section**

(a) **E-Fields of Charge Shells:** Remember Gauss's law: the total charge inside determines the electric flux density coming out of an enclosed surface. In this problem there is also spherical symmetry with respect to electric flux density and, hence, the electric field. For every $nR < r < (n+1)R$ with odd n, the net total charge within a sphere of radius r is +1 C. Thus, the field will be identical to that of a +1 C point charge centered at the origin:

$$\vec{E} = \frac{+1\text{ C}}{4\pi\epsilon_0 R^2}\hat{a}_r$$

For every $nR < r < (n+1)R$ with even n, the net total charge within a sphere of radius r is 0 C; there are no fields within this region. Thus, a rough sketch of field magnitude would resemble:



(b) **Gauss's Law:** If we draw a closed spherical surface around the origin of distance $R + \delta r$ (where $\delta r < 1\text{m}$), we can see that we will enclose a total of $+R^2$ charge. Thus, our electric field is

$$\vec{E} = \frac{R^2}{4\pi\epsilon_0 (R+\delta r)^2}\hat{a}_r$$

which becomes a constant

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\hat{a}_r$$

in the limit of large R.

(c) **Field Properties:**

(i) B)oth – curl and divergence are zero

(ii) N)either – curl and divergence are non-zero

(iii) E)-field – divergence is non-zero, curl is zero

(iv) H)-field – curl is non-zero, divergence is zero

(d) **Operators in Other Coordinate Systems:** In the spherical coordinate system, the unit vectors used to decompose the field components change direction as a function of observation point. The gradient, curl, and divergence formulas differ from their simpler Cartesian operators (since the Cartesian unit vectors do not change orientation depending on observation point).

Work-out Problem Section

(3) **Capacitance of a MOSFET:** We use the definition $C = Q/V$. If there is a charge density of $+\rho_S$ on the gate, then the total charge is $Q = A\rho_S$. If the flux density beneath the gate is $-\rho_S \hat{a}_z$, then the fields in each region are:

$$\mathrm{SiO_2}: \vec{E}_1 = -\frac{\rho_S}{\epsilon_1}\hat{a}_z \qquad \text{Si Substrates}: \vec{E}_2 = -\frac{\rho_S}{\epsilon_2}\hat{a}_z$$

Recall that normal $\vec{D}$ components are equal across dielectric boundaries. To get voltage, we simply integrate these fields from bottom to top:

$$\begin{aligned} V &= -\int_A^B \vec{E} \cdot d\vec{L} \\ &= -\int_0^{d_2} \vec{E}_2 \cdot dz\hat{a}_z - \int_{d_2}^{d_1+d_2} \vec{E}_1 \cdot dz\hat{a}_z \\ &= \rho_S \left[\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} \right] \end{aligned}$$

Thus,

$$C = \frac{Q}{V} = \frac{A\rho_S}{\rho_S \left[\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} \right]} = \frac{A\epsilon_1\epsilon_2}{\epsilon_2 d_1 + \epsilon_1 d_2}$$

(4) **Cylinder Current:** Biot-Savart law for surface current density:

$$\vec{H} = \int_A \frac{\vec{K} \times (\vec{r} - \vec{r}')dA}{4\pi|\vec{r} - \vec{r}'|^3}$$

When all of the appropriate numbers are plugged into this equation, the $\vec{H}$-field integral becomes

$$\vec{H}(x,y,z) = \int_{-L/2}^{L/2} \int_0^{2\pi} \frac{K_0(-\sin\phi\hat{a}_x + \cos\phi\hat{a}_y) \times ([x - R\cos\phi]\hat{a}_x + [y - R\sin\phi]\hat{a}_y + [z - z']\hat{a}_z)\, Rd\phi\, dz'}{4\pi\left[(x - R\cos\phi)^2 + (y - R\sin\phi)^2 + (z - z')^2\right]^{\frac{3}{2}}}$$

This integral is ready for numerical calculation. No further simplification or evaluation is required for full credit.

(5) **Crazy Continuity:** Recall that current is equal to the current density integrated over the cross-section area. The total current carried by the sheet is WK_0. The total current carried by the square tube is $W^2 J_1$. The total current carried by the cylinder is $\frac{\pi}{4}W^2 J_2$. Since all of these must equal I, we have the following results:

$$K_0 = \frac{I}{W} \qquad J_1 = \frac{I}{W^2} \qquad J_2 = \frac{4I}{\pi W^2}$$