ECE 3025: Electromagnetics

SOLUTIONS TO TEST 3 (Fall 2003)

(1) **Field Properties:**

(a) Neither – These fields circulate ($\nabla\times \neq 0$) *and* start/stop on sources ($\nabla\cdot \neq 0$).

(b) Both – A constant field has zero curl and zero divergence: a valid magnetic or electric field.

(c) Magnetic – These fields only circulate ($\nabla\times \neq 0$).

(d) Electric – These fields do not circulate ($\nabla\times = 0$) but do start/stop on sources/charges ($\nabla\cdot \neq 0$).

(2) **Thick Wire Current:**

(a) Ampere's Law states that the line integral of magnetic field around a closed path is equal to the total current through the enclosed surface.

$$\oint_L \vec{H}\cdot d\vec{L} = \underbrace{\iint_A \vec{J}\cdot d\hat{n}}_{I}$$

We also invoke our old symmetry arguments: 1) magnetic field circulates around the current in the z-direction so that it will only have a ϕ-component and 2) this problem is symmetrical with respect to z and ϕ so there will be only a ρ dependency. Thus, we may write:

$$\vec{H} = H_\phi(\rho)\hat{a}_\phi$$

Immediately, we can solve the left-hand side of Ampere's Law for a circle around the current:

$$\oint_L \vec{H}\cdot d\vec{L} = \int_0^{2\pi} [H_\phi(\rho)\hat{a}_\phi]\cdot[\rho d\phi \hat{a}_\phi] = 2\pi\rho H_\phi(\rho)$$

This is just the magnetic field strength, $H_\phi(\rho)$, multiplied by the circumference of the circular path with radius ρ. (You may not even need to do the math for this one – just evaluate it geometrically). The right-hand side may be evaluated geometrically as well:

Enclosed Current for $\rho < R$: $I = \pi\rho^2 J_0$

Enclosed Current for $\rho > R$: $I = \pi R^2 J_0$

Thus, the final answer for the magnetic field for all of space is

$$\vec{H} = \begin{cases} \frac{J_0\rho}{2}\hat{a}_\phi & \text{for } \rho < R \\ \\ \frac{J_0 R^2}{2\rho}\hat{a}_\phi & \text{for } \rho > R \end{cases}$$

(b) Recall that $\vec{J} = \sigma\vec{E}$. Thus,

$$\vec{E} = \frac{J_0}{\sigma}\hat{a}_z$$

(c)

$$\text{Inside Wire: } \nabla \times \vec{H} = \nabla \times \frac{J_0 \rho}{2} \hat{a}_\phi = J_0 \hat{a}_z$$

$$\text{Outside Wire: } \nabla \times \vec{H} = \nabla \times \frac{J_0 R^2}{2\rho} \hat{a}_\phi = 0$$

Note: This was by far the most difficult question on the test and, as such, part (a) was originally the entire 30-point problem. To help out, I split this problem up into 3 portions and gave lots of partial credit, taking into consideration that part (c) and part (a) were related. In fact, part (c) provided a nice hint for many students that were lost about how to start part (a).

(3) **MOSFET Current:**

(a) Working from the surface-integral form of the Biot-Savart law on the equation sheet:

$$\vec{H}(\vec{r}) = \iint\limits_S \frac{\vec{K}(\vec{r}') \times (\vec{r} - \vec{r}')dS}{4\pi|\vec{r} - \vec{r}'|^3} = \frac{K_0}{4\pi} \int\limits_{-\frac{L}{2}}^{\frac{L}{2}} \int\limits_{-\frac{W}{2}}^{\frac{W}{2}} \frac{\hat{a}_x \times [(x - x')\hat{a}_x + (y - y')\hat{a}_y + z\hat{a}_z] \; dx' \, dy'}{[(x - x')^2 + (y - y')^2 + z^2]^{\frac{3}{2}}}$$

No further simplifications are needed to get full credit. If you arrived at this step and continued to simply incorrectly, it did not count against you.

(b) There would be some sort of return current from drain to source that would also contribute some magnetic fields. (The planar current sheet alone does not follow the continuity equation.)

(4) **TV Buttons:**

(a) Contrast – Amplifying the intensity signal to the choke will cause bigger modulations in the electron beam intensity.

(b) Sharpness – Changing this voltage offset in the electrostatic lens will change the focal point of the electron beam.

(c) Width – The current in the vertical coils leads to vertical magnetic *flux*, which pushes the electron beam to the right or left, depending on the polarity. *Remember: the magnetic force is always perpendicular to the magnetic flux* **and** *the motion of charges.* Thus, amplifying the signal to the vertical coils will increase the total right-to-left swing of the electron beam.

(d) Horizontal Offset – Adding a DC offset to the vertical coil signal will shift the beam left or right, depending on the polarity of the offset.

(e) Height – The current in the horizontal leads to horizontal magnetic *flux*, which pushes the electron beam up or down, depending on the polarity. Thus, amplifying the signal to the horizontal coils will increase the total up-to-down swing of the electron beam.

(f) Horizontal Offset – Adding a DC offset to the horizontal coil signal will shift the beam up or down, depending on the polarity of the offset.