ECE 3025: Electromagnetics
Solutions to TEST 3 (Spring 2008)

(1) **Short Answer Section** (20 points)

(a) homogeneous, linear, isotropic, source-free

(b) diamagnetism

(c) ferromagnetism

(d) paramagnetism

(e) ferrimagnetism

(f) super-paramagnetism

(g) anti-ferrimagnetism

(2) **Continuity Equation:**

$$
\begin{aligned}
\nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\
\nabla \cdot (\nabla \times \vec{H}) &= \nabla \cdot (\vec{J} + \frac{\partial \vec{D}}{\partial t}) \\
0 &= \nabla \cdot \vec{J} + \frac{\partial \nabla \cdot \vec{D}}{\partial t} \\
0 &= \nabla \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} \\
\nabla \cdot \vec{J} &= -\frac{\partial \rho_v}{\partial t}
\end{aligned}
$$

In words, the most succinct way to express this relationship is "Conservation of Charge". If the divergence (i.e. "sourciness") of a current field is positive, that means that positive changes are being depleted in that region of space; an increasingly negative charge should be left behind. If the divergence is negative, that means that negative charges are being taken from the local region of space; an increasingly positive charge should be left behind.

(3) **Rail Gun Magnetic Fields:**

(a)

$$\begin{aligned}
\vec{B}(0,D,0) &= \overbrace{\mu\int_0^D \frac{Id\vec{L}\times(\vec{r}-\vec{r}')}{4\pi||\vec{r}-\vec{r}'||^3}}^{\text{Current to Projectile}} + \overbrace{\mu\int_0^D \frac{-Id\vec{L}\times(\vec{r}-\vec{r}')}{4\pi||\vec{r}-\vec{r}'||^3}}^{\text{Current from Projectile}} \\
&= \mu\int_0^D \frac{Idy'\hat{y}\times(D\hat{y}+\frac{d}{2}\hat{x}-y'\hat{y})}{4\pi||D\hat{y}+\frac{d}{2}\hat{x}-y'\hat{y}||^3} + \mu\int_0^D \frac{-Idy'\hat{y}\times(D\hat{y}-\frac{d}{2}\hat{x}-y'\hat{y})}{4\pi||D\hat{y}+\frac{d}{2}\hat{x}-y'\hat{y}||^3} \\
&= -\mu\hat{z}\int_0^D \frac{Idy'\frac{d}{2}}{4\pi((D-y')^2+\frac{d^2}{4})^{\frac{3}{2}}} - \mu\hat{z}\int_0^D \frac{Idy'\frac{d}{2}}{4\pi((D-y')^2+\frac{d^2}{4})^{\frac{3}{2}}} \\
&= -\frac{I\mu d}{4\pi}\hat{z}\int_0^D \frac{dy'}{((D-y')^2+\frac{d^2}{4})^{\frac{3}{2}}}
\end{aligned}$$

(b) Surface current density vector on the rails is

$$\vec{K} = \pm\frac{I}{h}\hat{y}$$

(c) The Ohmic resistance of the two rails will increase as the projectile slides down the gun; current through this leg of the circuit will decrease.

(d) The two rails will act like transmission lines. If it is not matched to the source, there will be reflections, which lead to intermittent forces on the projectile.

(4) **Inductive Toothbrush Charger:**

(a) Charging is optimized when the capacitor impedance is adjusted to cancel the self-inductive impedance, just like a 13.56 MHz RFID tag. This occurs at

$$C = \frac{1}{4\pi^2 f^2 L}$$

Conveniently, this value works both for the toothbrush-side part of the circuit *and* the source-side circuit.

(b) When $M \approx L$ in a two-loop system like this, it implies that *all* of the magnetic flux through each loop is shared. In other words, of all the flux going through one loop, *none* of it travels around the second loop. This will only happen if the two coils are extremely close to one another.

(c) There source resistance R_s and the Thevenin equivalent resistance Z_{Th} are the two power-absorbing elements in the circuit. Assuming the reactive components have been tuned out, the simplified Thevenin equivalent resistance is

$$Z_{Th} = \frac{4\pi^2 f^2 M^2}{R_b}$$

Note that many people did not simplify Z_{Th}, opting to use the full complex expression. If the efficiency calculation was set up properly, I did not deduct points for this ... or for leaving the resulting complicated expression unevaluated.

Efficiency, η, is power delivered to the dead battery (effectively the real part of Z_{Th}) compared to the total power in the system. Thus,

$$\eta = \frac{I^2 \text{Real}\{Z_{Th}\}}{I^2 \text{Real}\{R_s + Z_{Th}\}} = \frac{4\pi^2 f^2 M^2}{R_s R_b + 4\pi^2 M^2 f^2}$$

Efficiency approaches 1 as the frequency increases – a result that should confirm our intuition about this system.