Homework 2: ECE 4370

Radiation and Line Currents

1. In class, we presented expressions for vector magnetic potential, magnetic field, and electric field for a Hertzian dipole. Show that the vector magnetic potential for the Hertzian dipole that we use in class satisfies the wave equation:

   $$\left(\nabla^2 + k^2\right)\tilde{\vec{A}} = -\mu\tilde{\vec{J}}$$

   when the current density is a z-directed impulse of current at the origin.

   (**10 points**)

2. Given the vector magnetic potential solution for a Hertzian dipole, derive the exact (near and far field) solution for E-field and H-field from their vector calculus definitions. (**10 points**)

3. Calculate the ideal, far-field gain pattern and radiation resistance for the "short dipole" of length L with tapered line current:

   $$\tilde{I}(z) = I_0\left(1 - \frac{2|z|}{L}\right)u\left(\frac{L}{2} - |z|\right) \quad \text{for } L << \lambda$$

   How does this gain pattern compare to the Hertzian dipole? (**10 points**)