Homework 2: ECE 4370

Radiation and Line Currents

1. In class, we presented expressions for vector magnetic potential, magnetic field, and electric field for a Hertzian dipole. Show that the vector magnetic potential for the Hertzian dipole that we use in class satisfies the wave equation:

\[
\left( \nabla^2 + k^2 \right) \tilde{A} = -\mu \tilde{J}
\]

when the current density is a z-directed impulse of current at the origin.

(10 points)

2. Given the vector magnetic potential solution for a Hertzian dipole, derive the exact (near and far field) solution for E-field and H-field from their vector calculus definitions. (10 points)

3. Calculate the ideal, far-field gain pattern and radiation resistance for the “short dipole” of length L with tapered line current:

\[
\tilde{I}(z) = I_0 \left( 1 - \frac{2|z|}{L} \right) \mu \left( \frac{L}{2} - |z| \right) \quad \text{for} \ L << \lambda
\]

How does this gain pattern compare to the Hertzian dipole? (10 points)