

## Homework 7

ECE 4370

Problem 1:

Assuming an ideal field distribution with uniform phase from the  $TE_{10}$  mode of a rectangular waveguide has been propagated to the aperture of a horn antenna with width  $A$  and height  $B$ , calculate the directivity pattern and ideal peak gain (assuming 100% radiation efficiency) in terms of  $A$ ,  $B$ , and  $\lambda$ . (10 points)

Problem 2:

You must design a 4 GHz horn antenna with maximum peak gain, fed by a standard rectangular waveguide ( $a = 2b = \lambda/2$ ). The maximum length of the flared horn antenna,  $R_p$ , must not exceed 50 cm. Provide an engineering diagram, fully dimensioned, for your company's metal shop to produce. (10 points)

Solution to Homework 7

Problem 1:

For TE<sub>10</sub> Horn Antenna

$$\tilde{E}_0(x', y', 0) = E_0 \cos\left(\frac{\pi x'}{a}\right)$$

KHIT:  $\tilde{E}_0(r, \theta, \phi) = j \frac{E_0 k \cos\theta}{4\pi r} \exp(-jkr) \cdot$

$$\int_{-b/2}^{+b/2} dy' \int_{-a/2}^{a/2} dx' \cos\left(\frac{\pi x'}{a}\right) \exp(jk[\sin\theta \cos\phi x' + \sin\theta \sin\phi y'])$$

$$= j \frac{E_0 k \sin(k \sin\theta b \sin\phi / 2)}{2\pi r k \sin\theta \sin\phi} \int_{-a/2}^{a/2} dx' \cos\left(\frac{\pi x'}{a}\right) \exp(jk \sin\theta \cos\phi x')$$

$$= j \frac{a E_0 \sin(k \sin\theta b \sin\phi / 2) \exp(-jkr)}{r \sin\theta \sin\phi [\pi^2 - a^2 k^2 \sin^2\theta \cos^2\phi]} \cdot \cos\left(\frac{a k \sin\theta \cos\phi}{2}\right)$$

$$\vec{S}_{iso} = \frac{1}{4\pi r^2} \iint \frac{E_0(x', y', 0)}{2\eta} dx' dy' = \frac{E_0^2 ab}{16\eta \pi r^2}$$

$$D(\phi, \theta) = \frac{4a\pi \cot^2\theta \sin^2(bk \sin\theta \sin\phi / 2) \cos^2\left(\frac{a k \sin\theta \cos\phi}{2}\right)}{b \sin^2\phi [\pi^2 - a^2 k^2 \sin^2\theta \cos^2\phi]^2}$$

Peak Gain  $D(0, 0) = \frac{4\pi}{\lambda^2} (ab)$

Problem 2:

Horn Antenna Calculator		
Frequency	4000	MHz
Wavelength	0.075	m
Waveguide Width, a	0.038	m
Waveguide Width, b	0.019	m
Flare Length, Rp	0.500	m
Aperture Width, A	0.340	m
Aperture Width, B	0.276	m
H-Taper Depth, RH	0.562	m
E-Taper Depth, RE	0.536	m
Ideal Peak Gain	23.2	dBi