Homework 3: ECE 4370

Solutions

1. Recall our procedure for calculating fields from a line-current antenna: find the vector magnetic potential first and then translate into a Poynting vector:

$$\text{Power Density (W/m}^2\text{):} \quad = \frac{k^2 \eta \sin^2\theta}{2\mu^2} |\tilde{A}_z|^2$$

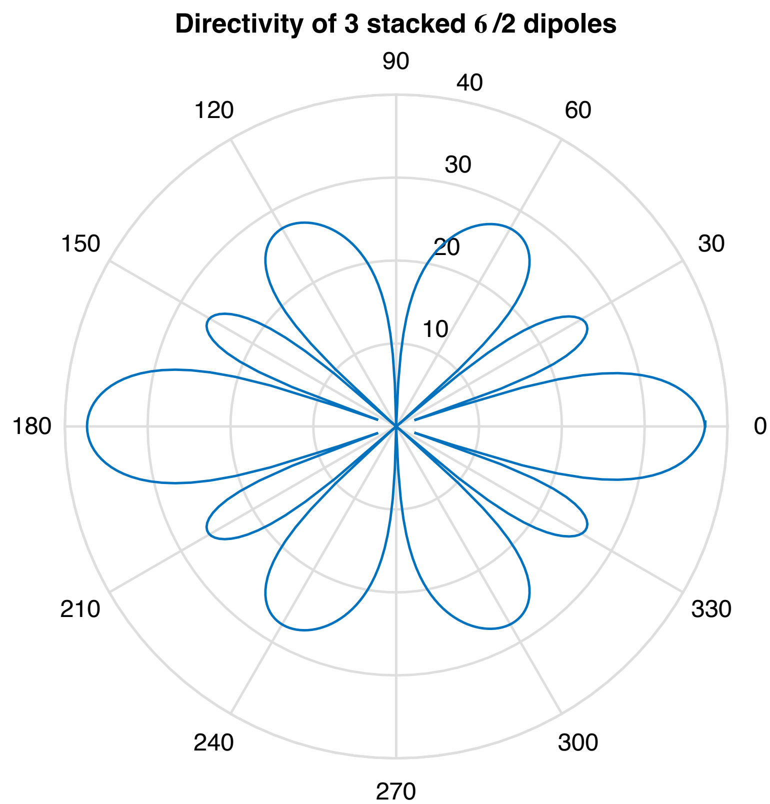
The integral for the stacked array of dipoles is just the sum of half-wave dipole integrals:

$$\begin{aligned}
|\tilde{A}_z(\vec{r})| &= \frac{\mu}{4\pi r} \left| \int_{-\lambda/4}^{(2N-1)\lambda/4} I|\cos(kz')|\exp(jkz'\cos\theta)\; dz' \right| \\
&= \frac{\mu}{4\pi r} \left| \sum_{n=0}^{N-1} (-1)^n \int_{(2n-1)\lambda/4}^{(2n+1)\lambda/4} I\cos(kz')\exp(jkz'\cos\theta)\; dz' \right| \\
&= \frac{\mu}{4\pi r} \left| \sum_{n=0}^{N-1} \frac{2I\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta} \exp(jn\pi\cos\theta) \right| \\
&= \frac{\mu}{2\pi r} \frac{I\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta} \underbrace{\left| \sum_{n=0}^{N-1} \exp(jn\pi\cos\theta) \right|}_{\sin(N\pi/2\cdot\cos\theta)/\sin(\pi/2\cdot\cos\theta)}
\end{aligned}$$

Now we have enough information to plot the power density of waves traveling away from the current source:

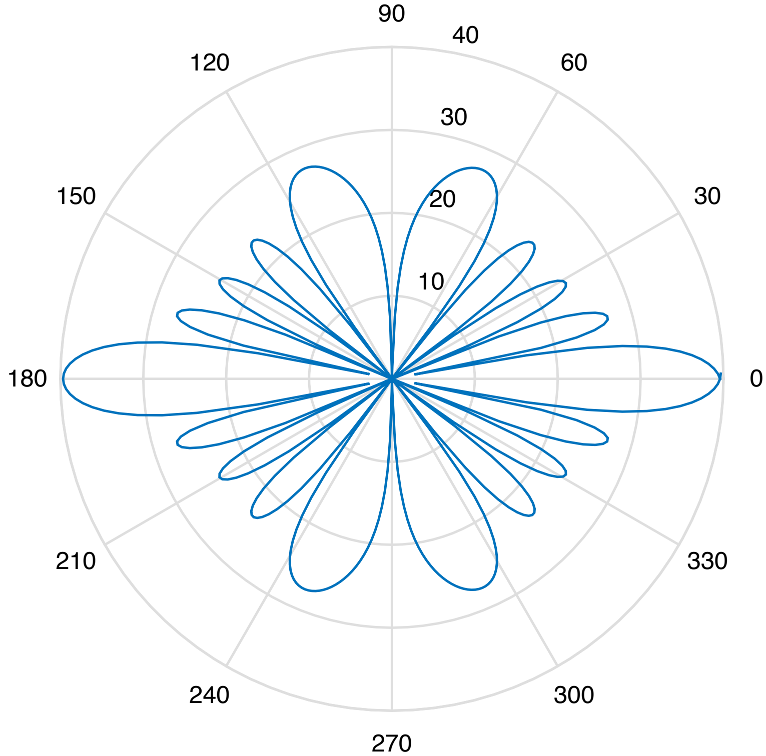
$$\begin{aligned}
\text{Power Density (W/m}^2\text{):} \quad &= \frac{k^2 \eta \sin^2\theta}{2\mu^2} |\tilde{A}_z|^2 \\
&= \frac{I^2\eta}{8\pi^2 r^2} \underbrace{\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}}_{\text{dipole gain pattern}} \underbrace{\frac{\sin^2\left(\frac{N\pi}{2}\cos\theta\right)}{\sin^2\left(\frac{\pi}{2}\cos\theta\right)}}_{\text{array pattern}}
\end{aligned}$$

See the attached Matlab code for computations. The following shows the code output for cases of 3, 5, 7, and 9 dipoles. Note that “up” is the z-axis, in contrast to Matlab’s typical polar plot convention.



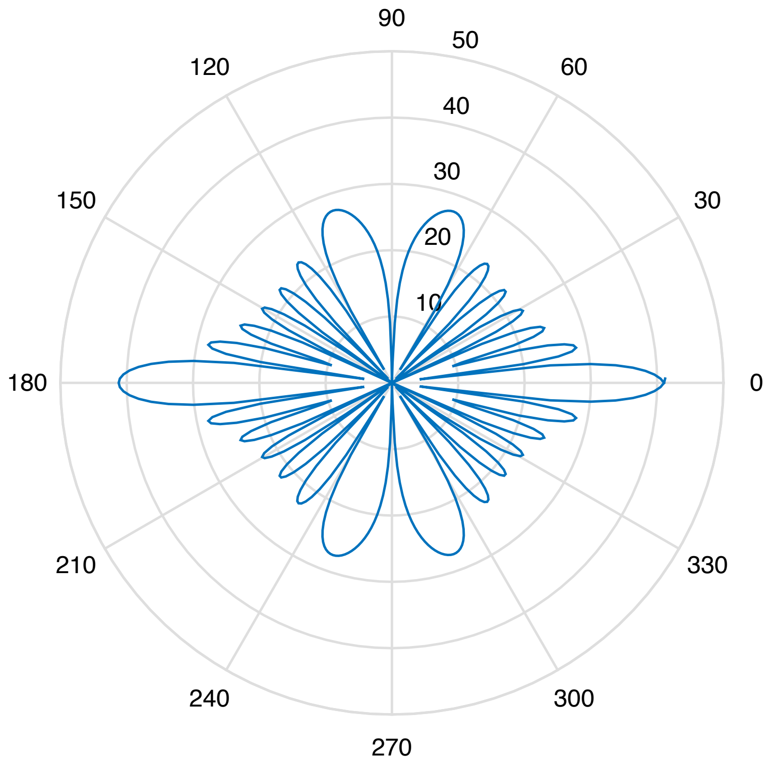

```
Half-wave dipole array of 3 element(s)
 Element input current 3.0 A
 ----------------------------------------------
 Total radiated power:    100.0 W (20.0 dBW)
 Peak gain:               5.4 (7.3 dBi)
 Half-power beadwidth:    17.1 deg
 Side-lobe level:         9.8 dB
 Radiation resistance:    22.2 Ohms
```

Directivity of 5 stacked 6 /2 dipoles



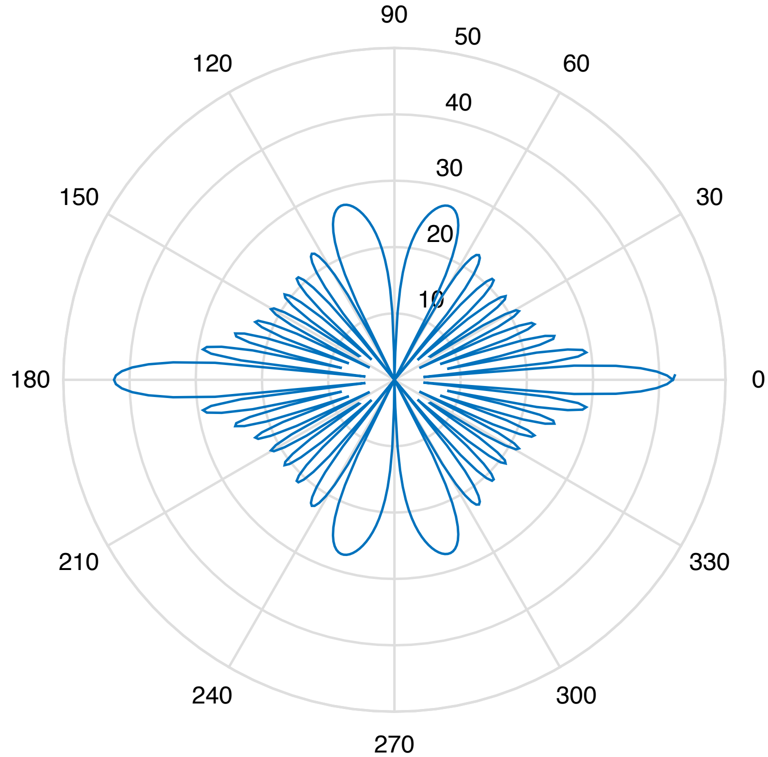

```
Half-wave dipole array of 5 element(s)
 Element input current 5.0 A
 ----------------------------------------------
Total radiated power:   161.5 W (22.1 dBW)
 Peak gain:              9.3 (9.7 dBi)
 Half-power beadwidth:   9.9 deg
 Side-lobe level:        12.2 dB
 Radiation resistance:   12.9 Ohms
```

Directivity of 7 stacked 6 /2 dipoles




```
Half-wave dipole array of 7 element(s)
 Element input current 7.0 A
 ----------------------------------------------
 Total radiated power:   222.4 W (23.5 dBW)
 Peak gain:              13.2 (11.2 dBi)
 Half-power beadwidth:   8.1 deg
 Side-lobe level:        12.9 dB
 Radiation resistance:   9.1 Ohms
```

Directivity of 9 stacked 6 /2 dipoles




```
Half-wave dipole array of 9 element(s)
 Element input current 9.0 A
 ----------------------------------------------
 Total radiated power:    283.0 W (24.5 dBW)
 Peak gain:               17.2 (12.3 dBi)
 Half-power beadwidth:    6.3 deg
 Side-lobe level:         13.1 dB
 Radiation resistance:    7.0 Ohms
```

Note that the product of the half-power beamwidth and the peak linear gain is (almost) always conserved.

```
% DipoleArray.m (ver 1.0)
% Author:  Greg Durgin                  Date: 27 September 2011
%
% Compute the pattern and electrical parameter for a stack of
% N vertical half-wave dipoles.
clear all; close all;   % initialize

% User-defined inputs
I = 1;                  % current amplitude (Amps)
N = 20;                 % number of stacked half-wave, in-phase dipoles
M = 200;                % number of elevation points to plot
dB_min = -30;           % minimum gain to plot in polar coordinates

% Initialized variables
M = 2*ceil(M/2);        % ensure M is even
mu_0 = 4*pi*1e-7;       % permeability of free space (H/m)
ep_0 = 8.85e-12;        % permittivity of free space (F/m)
eta = (mu_0/ep_0)^.5;   % impedance of free space (Ohms)
theta = (0:M)/M * pi;   % generate range of elevation angles (rad)

% Generate power pattern
P = I^2*eta/8/pi^2*cos(pi/2*cos(theta)).^2./sin(theta).^2 .* ...
    (sin(N*pi*cos(theta+1e-8))./sin(pi*cos(theta+1e-8))).^2;
P(1) = 0; P(end)=0;     % zero out nulls due to singularity
P_tot = sum(P.*sin(theta))*pi/(M+1)*2*pi;   % radiated power (W/m^2)
D = 4*pi*P/P_tot;                           % compute directivity

% Plotting functions
theta_p = (0:(2*M+1))/2/M * 2*pi;       % index with Matlab polar coords
D = [ fliplr(D(1:M/2)) D(1:M/2) D(M/2+1:end) fliplr(D(M/2+1:end)) ];
DdB = max(10*log10(abs(D)),dB_min)-dB_min; % compute log directivity (dB)
polar(theta_p,DdB);                     % plot the directivity (dB)
if N == 1
    title('Directivity of a \lambda/2 dipole');
else
    title(sprintf('Directivity of %i stacked \\lambda/2 dipoles', N));
end;
axis off;

% Radiation parameter computation and display
Gpeak = max(D);
Rrad = 2*P_tot/I^2;
hpbw = sum(D(1:M+1) >= Gpeak/2)/M*pi;      % calculate half-power BW
ind = D(2:(M/2))<D(1:(M/2-1));
SLL = max(D(([1 ind] - [ind 0])>0))/Gpeak; % get local maxima in directivity
fprintf('\n\n Half-wave dipole array of %i element(s)', N);
fprintf('\n Input current %2.1f A', I );
fprintf('\n ---------------------------------------------');
fprintf('\n Total radiated power:   %3.1f W (%2.1f dBW)', ...
    P_tot, 10*log10(P_tot) );
fprintf('\n Peak gain:              %3.1f (%2.1f dBi)', ...
    Gpeak, 10*log10(Gpeak) );
fprintf('\n Half-power beadwidth:   %2.1f deg', hpbw*180/pi );
fprintf('\n Side-lobe level:        %2.1f dB', -10*log10(SLL) );
fprintf('\n Radiation resistance:   %3.1f Ohms', Rrad );
fprintf('\n\n');
```

2. From the Friis formula:

$$P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi R)^2}$$

We have:

$P_r = -85dBm = 3.162 \times 10^{-9} mW$

$P_t = 200mW$

$G_t = 12dBi = 15.85$

$G_r = -2dBi = 0.63$

$\lambda = \frac{3}{7} m$

Thus, R=27.1km