Name: ______________________

GTID: ______________________

ECE 4370: Antenna Engineering
TEST 1 (Fall 2016)

- Please read all instructions before continuing with the test.
- This is a **closed** notes, **closed** book, **closed** friend, **open** mind test. On your desk you should only have writing instruments and a calculator. **No internet-enabled devices.**
- Show all work. (It helps me to give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.
- Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.
- All necessary mathematical formulas are included either in the problem statements or the last page of this test.
- You have 80 minutes to complete this examination. When the proctor announces a "last call" for examination papers, he will leave the room in 5 minutes. The fact that the proctor does not have your examination in hand will not stop him.
- I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.
- Have a nice day!

Pledge Signature: ______________________________________

I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.

1. **Directivity:** You possess two identical 200-Ω antennas with 3 dBi peak gain, pointed toward one another and separated by 100m of free-space. To one antenna – the transmitter – you connect a 300 MHz, 1-Volt source with 50-Ω impedance to the antenna. To the other antenna – the receiver – you connect a 50-Ω resistive load. Answer the following questions based on this scenario (**30 points**).

   (a) What is the magnitude of the voltage across the 50-Ω load resistor at the receiver? (**15 points**)

   (b) How much *total* power is the voltage *source* providing to this system in dBm? (**5 points**)

   (c) What is the total loss of the system in dB, comparing the power delivered to the load resistor to the power provided by the source in (b)? (**10 points**)

2. **Mystery Antenna:** Below is the far-field expression for the radiated Poynting vector for an antenna operating at 5 GHz. This antenna radiates an omnidirectional pattern along the azimuthal plane ($\theta = 90°$) over the interval $\frac{\pi}{2} - \theta_o \leq \theta \leq \frac{\pi}{2} + \theta_o$. Answer the following questions based on this distribution. (**45 points**)

$$\vec{\mathrm{S}}_{\mathrm{av}}(r, \theta, \phi) = \frac{10\,\mathrm{mW}}{4\pi r^2}\hat{r} \quad \text{for } \frac{\pi}{2} - \theta_o \leq \theta \leq \frac{\pi}{2} + \theta_o, \qquad \text{0 elsewhere}$$

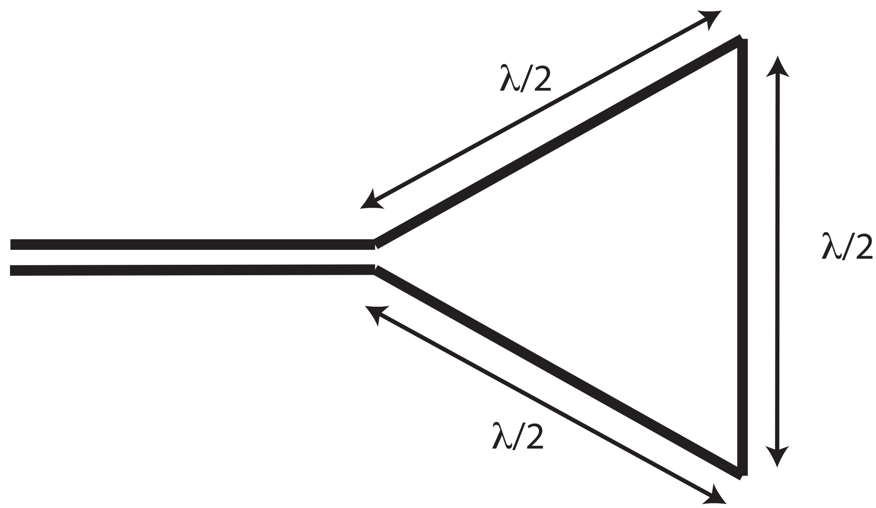
(a) Write expressions for the E-field and H-field phasor-form solutions for the radiated waves in three-dimensions. Assume that the E-field polarization is in the $\hat{\theta}$ direction. (**20 points**)

(b) What is the half-power beamwidth with respect to elevation (in degrees) of this antenna? (**5 points**)

(c) What is the peak directivity of this antenna? (**10 points**)

(d) What is the electromagnetic aperture of this antenna along the direction of peak directivity? (assume an ideal, lossless antenna) (**10 points**)

3. **Triangle Antenna:** Below is a diagram of a transmission line-fed antenna, consisting of three half-wavelength sides arranged in the shape of an equilateral triangle. Answer the following questions based on this distribution. (**25 points**)



(a) Estimate and sketch the amplitude distribution of current standing waves on this structure. Be sure to mark relative polarity of the current with arrows. (**20 points**)

(b) If this antenna is used at 100 MHz, how far away from the antenna does observation of the "far field" start? (**5 points**)

(c) **Bonus:** What is the radiation resistance of this antenna? (**+5 points**)

Cheat Sheet

$$\lambda f = c \qquad c = 3\times 10^8 \text{ m/s} \qquad \mu_o = 4\pi\times 10^{-7} \text{ H/m} \qquad \epsilon_o = 8.85\times 10^{-12} \text{ F/m} \qquad k = \frac{2\pi}{\lambda}$$

$$\text{Directivity} = \frac{\text{Radiated Power Density in } (\phi,\theta)}{\text{Average Isotropically Radiated Power Density}} = \frac{\left\|\vec{\mathrm{S}}_{av}(r,\theta,\phi)\right\|}{P_T/(4\pi r^2)}$$

$$D(\phi,\theta) = \eta_{rad}\, G(\phi,\theta) \qquad P_T = \frac{1}{2} I^2 R_{rad} \qquad D = \frac{4\pi}{\lambda^2} A_{em}$$

$$\int_0^{2\pi}\int_0^{\pi} D(\phi,\theta)\,\sin\theta\, d\phi\, d\theta = 4\pi \ \text{ steradians}$$

$$\tilde{A}_z(\vec{\mathrm{r}}) = \frac{\mu}{4\pi r}\exp(-jkr)\int\int_{-\infty}^{+\infty}\int \tilde{J}_z(x',y',z')\exp(+jk[x'\sin\theta\cos\phi + y'\sin\theta\sin\phi + z'\cos\theta])\ dx'\,dy'\,dz'$$

$$\text{Line Current:} \qquad \tilde{A}_z(r,\theta,\phi) = \frac{\mu}{4\pi r}\exp(-jkr)\int_{-\infty}^{+\infty} \tilde{I}_z(z')\exp(+jkz'\cos\theta)\ dz'$$

$$\tilde{\vec{\mathrm{H}}}(r,\phi,\theta) = \frac{1}{\mu}\nabla\times(\tilde{A}_z\hat{\mathrm{z}}) \approx \frac{jk\sin\theta}{\mu}\tilde{A}_z(r,\phi,\theta)\hat{\phi} \qquad \tilde{\vec{\mathrm{E}}}(r,\phi,\theta) = \frac{1}{j2\pi f\mu\epsilon}\nabla\times\nabla\times(\tilde{A}_z\hat{\mathrm{z}}) \approx \frac{jk\eta\sin\theta}{\mu}\tilde{A}_z(r,\phi,\theta)\hat{\theta}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 377\ \Omega \text{ for free space} \qquad \vec{\mathrm{S}}_{av} = \frac{1}{2}\text{Real}\left\{\tilde{\vec{\mathrm{E}}}\times\tilde{\vec{\mathrm{H}}}^*\right\} \approx \frac{1}{2}\left\|\tilde{\vec{\mathrm{E}}}\right\|\left\|\tilde{\vec{\mathrm{H}}}\right\|\hat{\mathrm{r}}$$

above approximations valid for far field, $r > D^2/\lambda$

Basic Voltage Formulas

$$P_L = \underbrace{\frac{4R_L R_A}{\left|\tilde{Z}_A + \tilde{Z}_L\right|^2}}_{\text{mismatch losses}} P_R \qquad V_A = \frac{2\left|\tilde{Z}_L\right|\sqrt{2R_A P_R}}{\left|\tilde{Z}_A + \tilde{Z}_L\right|}$$