ECE 4370: Antenna Engineering
TEST 2 (Fall 2012)

• Please read all instructions before continuing with the test.

• This is a **closed** notes, **closed** book, **closed** friend, **open** mind test. On your desk you should only have writing instruments and a calculator. **No internet-enabled devices.**

• Show all work. (It helps me to give partial credit.) Work all problems in the spaces below the problem statement. If you need more room, use the back of the page. DO NOT use or attach extra sheets of paper for work.

• Work intelligently – read through the exam and do the easiest problems first. Save the hard ones for last.

• All necessary mathematical formulas are included either in the problem statements or the last page of this test.

• You have 80 minutes to complete this examination. When the proctor announces a “last call” for examination papers, he will leave the room in 5 minutes. The fact that the proctor does not have your examination in hand will not stop him.

• I will not grade your examination if you fail to 1) put your name and GTID number in the upper left-hand blanks on this page or 2) sign the blank below acknowledging the terms of this test and the honor code policy.

• Have a nice day!

Pledge Signature: ___________________________________________

*I acknowledge the above terms for taking this examination. I have neither given nor received unauthorized help on this test. I have followed the Georgia Tech honor code in preparing and submitting the test.*
1. **Linear Antenna Arrays**: In the space below, sketch the magnitude for the array factor for each uniform linear antenna array. Assume that the array elements are placed along the y-axis and that all points of observation are in azimuth ($\theta = 90^\circ$). Only a sketch is required, but be sure to label where the main lobes and nulls of the array factor point. (30 points)

(a) $N = 2$ elements, $\beta = -90^\circ$, $d = \lambda/4$

(b) $N = 4$ elements, $\beta = 0^\circ$, $d = \lambda/2$

(c) $N = 2$ elements, $\beta = 45^\circ$, $d = \lambda/2$
2. **Broadband Antenna**: Your colleague is trying to design a broadband antenna and contemplates using a custom *saw-tooth dipole* antenna. The geometry of this antenna is specified by three parameters: tooth radius $\rho_0$, tooth angle $\alpha$, and number of teeth $N$. With these three parameters, the sawtooth dipole is a planar antenna whose area of metallization $(\rho, \phi)$ satisfies the following relationships:

$$-\frac{\pi}{4} + \alpha \left\lfloor \frac{\rho}{\rho_0} \right\rfloor \leq \phi \leq \frac{\pi}{4} + \alpha \left\lfloor \frac{\rho}{\rho_0} \right\rfloor \quad \text{or} \quad \frac{3\pi}{4} + \alpha \left\lfloor \frac{\rho}{\rho_0} \right\rfloor \leq \phi \leq \frac{5\pi}{4} + \alpha \left\lfloor \frac{\rho}{\rho_0} \right\rfloor \quad \text{and} \quad \rho \leq N\rho_0$$

where $\left\lfloor \right\rfloor$ is the floor (round down) operation. An example of an $N = 6$, $\alpha = 25^\circ$ sawtooth dipole is sketched below. Answer the following questions based on this antenna geometry. (15 points)

(a) Instead of hiring an external consultant to perform simulations and prototyping that would reveal the properties of this antenna, provide your colleague with an estimate of the antenna impedance. (10 points)

(b) What would limit the lower-frequency performance of this broadband antenna? (5 points)
3. **Directional Antennas:** Compare the designs of a horn and an axial-mode helix by answering the following questions. (40 points)

(a) Standard rectangular waveguide has a width-to-height ratio of 2:1 to strike a practical balance between low-loss and high bandwidth. Show mathematically why a very high-gain pyramidal horn (large flare distance $R_p$) must always taper this waveguide to a ratio of 1.22:1. (10 points)

(b) Given an aperture efficiency of 50% and the 1.22:1 ratio of width to height at the end of the horn, what would be the dimensions of a horn antenna ($A$, $B$, and $R_p$) that would produce 27 dBi of peak gain at 5 GHz? You may assume $R_p \gg \lambda$. (15 points)

(c) What would be the dimensions (total height and diameter) of an axial mode helix with 13-degree pitch angle that would achieve the same gain of 27 dBi at 5 GHz? (10 points)

(d) Ignoring your size estimates for these two antennas, why might an engineer select the helical antenna over the pyramidal horn antenna? (5 points)
4. **Electrically Small Antennas:** You are to design a smart phone antenna solution that operates at 700 MHz (newly allocated cellular band), 860 MHz (original cellular band), 1920 MHz (first digital cellular band), and 2.45 GHz (WiFi). Using your knowledge of the Chu-Harrington limit, would it be more space efficient to use two antennas on the handset (one for 700-860 MHz and one for 1920-2450 MHz) or a single antenna capable of receiving all 4 bands (700-2450 MHz)? (15 points)
Cheat Sheet

\( \lambda f = c \quad c = 3 \times 10^8 \text{ m/s} \quad \mu_o = 4\pi \times 10^{-7} \text{ H/m} \quad \epsilon_o = 8.85 \times 10^{-12} \text{ F/m} \quad k = \frac{2\pi}{\lambda} \)

\[
Q = \frac{\text{center frequency}}{\text{bandwidth}} \approx \frac{1}{(ka)^3} \quad a \text{ is diameter of bounding sphere}
\]

\[
\begin{align*}
N & \quad \text{Uniform Linear Array} \\
3 & \quad \text{d} \\
2 & \quad \text{1}
\end{align*}
\]

\[
|AF| = \left| \frac{\sin \left( \frac{\pi}{\lambda} N d \sin \phi - N \beta/2 \right)}{\sin \left( \frac{\pi}{\lambda} d \sin \phi - \beta/2 \right)} \right| \quad \text{for } \theta = 90^\circ \quad G_{\text{array}}(\theta, \phi) \propto G_{\text{element}}(\theta, \phi) \times |AF(\theta, \phi)|^2
\]

\[
\sin x \approx x \quad \text{for small } x
\]

Complimentary Antennas: \( Z_1 Z_2 = \frac{\eta^2}{4} \quad \eta = \sqrt{\frac{\mu}{\epsilon}} \)

Maximum Gain/Minimum Phase Distortion Criterion for Pyramidal Horn Antennas:

\[
A = \sqrt{3\lambda \rho_p + \frac{9\lambda^2}{16}} \quad B = \sqrt{2\lambda \rho_p + \frac{\lambda^2}{4}}
\]

\[
G_{\text{peak}} = \frac{4\pi A_{\text{em}}}{\lambda^2 - \eta_{\text{eff}}}
\]

Axial-Mode Helical Antenna Design (for operation over \( \frac{3\lambda}{4} \leq C \leq \frac{4\lambda}{3} \)):

\[
G_{\text{peak}} = 15N \frac{C^2 S}{\lambda^3} \quad R_{\text{rad}} \approx 140 \left( \frac{C}{\lambda} \right) \quad \text{HPBW} = \frac{52\lambda^{3/2}}{C\sqrt{NS}} \text{ degrees}
\]

\[
\begin{align*}
S & \quad \text{turn spacing} \\
N & \quad \text{number of turns} \\
C & \quad \text{circumference, } \pi D \\
\alpha & \quad \text{pitch angle, } \tan^{-1}(S/C) \\
h & \quad \text{total height, } NS
\end{align*}
\]